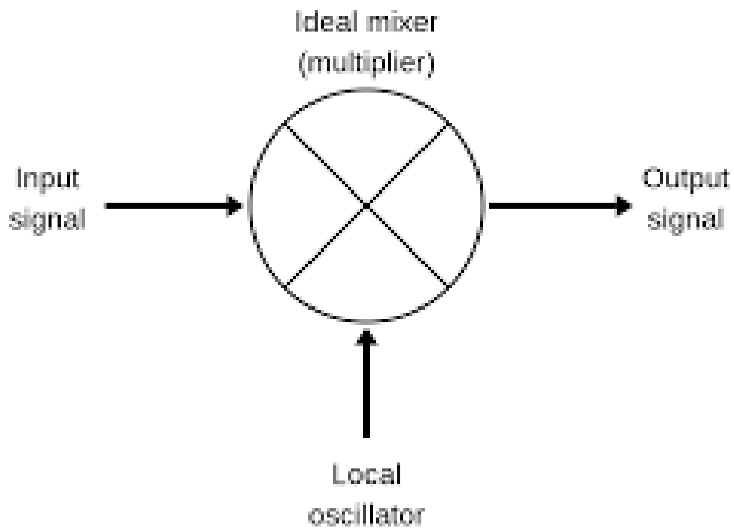
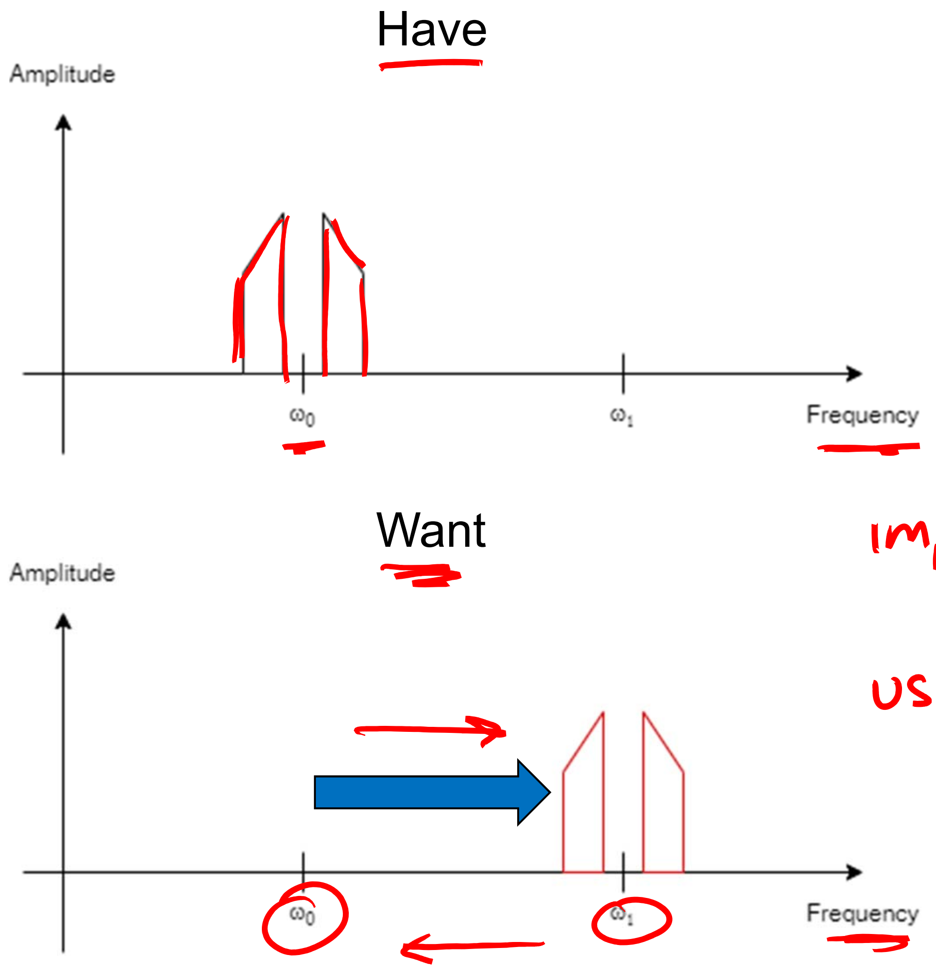
Frequency Mixing



Frequency Shift Operation



Frequency Domain
⇒ Fourier Transform

implemented MIXER
USED
Up-Conversion
Down-Conversion

We desire the ability to shift or translate signals in frequency.
How is this done?

Recall

- Euler Identity

ω_0 $-\omega_0$

$$\cos(\omega_0 t) = \frac{1}{2}\left(e^{j\omega_0 t} + e^{-j\omega_0 t}\right)$$

$A\cos(\Phi(t))$ $Ae^{j\Phi(t)}$

$\omega_0 t + \phi_0$

- Fourier Transform

$y(t)$ - time domain

$$Y(j\omega) = \frac{1}{T}\int_{-\frac{T}{2}}^{\frac{T}{2}} y(t)e^{-j\omega t}\,dt$$

$Y(j\omega)$ - Frequency Domain

The Math

Let $$x(t) = y(t) * e^{-j\omega_0 t}$$

Then $$X(j\omega) = \frac{1}{T}\int_{-\frac{T}{2}}^{\frac{T}{2}} \overbrace{y(t)e^{-j\omega_0 t}}^{x(t)} e^{-j\omega t}\, dt$$

$$X(j\omega) = \frac{1}{T}\int_{-\frac{T}{2}}^{\frac{T}{2}} y(t)e^{-j(\omega+\omega_0)t}\, dt$$

$\beta = \omega + \omega_0$

$$X(j\omega) = \frac{1}{T}\int_{-\frac{T}{2}}^{\frac{T}{2}} y(t)e^{-j\beta t}\, dt$$

$$X(j\omega) = Y(j\beta)$$

$$X(j\omega) = Y(j(\omega + \omega_0))$$

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The Math: Complex sinusoid input

Let $x(t) = y(t) * e^{-j\omega_0 t}$ $\qquad X(j\omega) = Y\big(j(\omega + \omega_0)\big)$

and $y(t) = e^{-j\omega_1 t}$ $\qquad Y(j\omega) = \delta\big(j(\omega - \omega_1)\big)$

$$\delta(x) = \begin{cases} 1, & x = 0 \\ 0, & otherwise \end{cases}$$

$$x(t) = e^{-j\omega_1 t} * e^{-j\omega_0 t}$$

$$x(t) = e^{-j(\omega_0 + \omega_1)t}$$

$$X(j\omega) = \begin{cases} 1, & \omega = \omega_0 + \omega_1 \\ 0, & ow \end{cases}$$

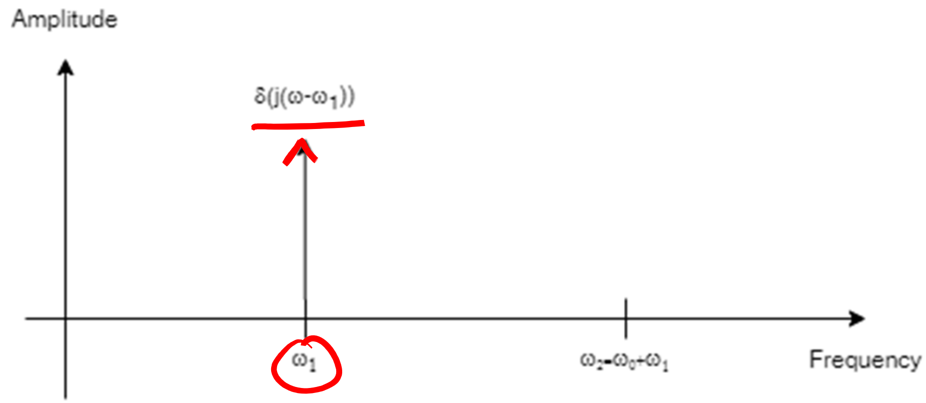
$$X(j\omega) = \delta\big(j(\omega - (\omega_o + \omega_1))\big)$$

Multiplication by a complex sinusoid in the time domain is a frequency shift in the frequency domain

Graphically

$\cancel{X(j\omega)}$ $Y(j\omega)$

$$y(t) = e^{-j\omega_1 t}$$

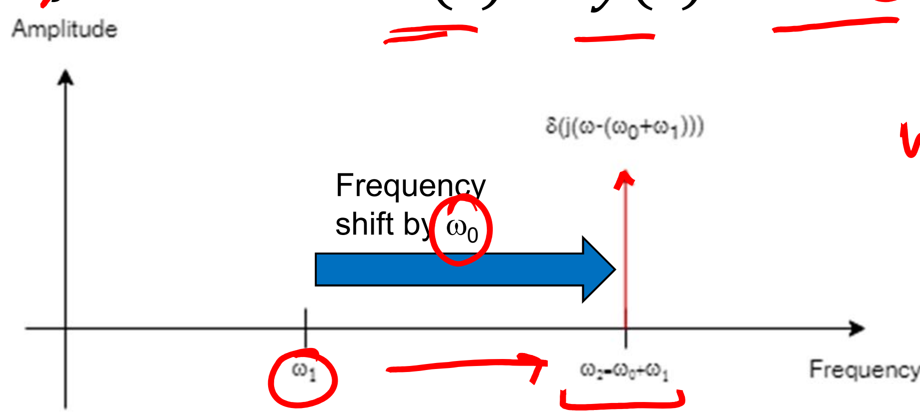


Multiplication in the time Domain = A Frequency Shift in the frequency domain.

$Y(\cancel{j}\omega)$ $X(j\omega)$

$$x(t) = y(t) * e^{-j\omega_0 t} = e^{-j(\omega_0+\omega_1)t}$$

$\omega_0 + \omega_1$



What about a “real” sinusoid?

$$x(t) = y(t) * \cos(\omega_0 t) \qquad \cos(\omega_0 t) = \frac{1}{2}\left(e^{j\omega_0 t} + e^{-j\omega_0 t}\right)$$

$$y(t) = e^{-j\omega_1 t}$$

$$x(t) = e^{-j\omega_1 t} * \cos(\omega_0 t)$$

$$x(t) = e^{-j\omega_1 t} * \frac{1}{2}\left(e^{j\omega_0 t} + e^{-j\omega_0 t}\right)$$

$$x(t) = \frac{1}{2}\left(e^{j(\omega_0 - \omega_1)t} + e^{-j(\omega_0 + \omega_1)t}\right)$$

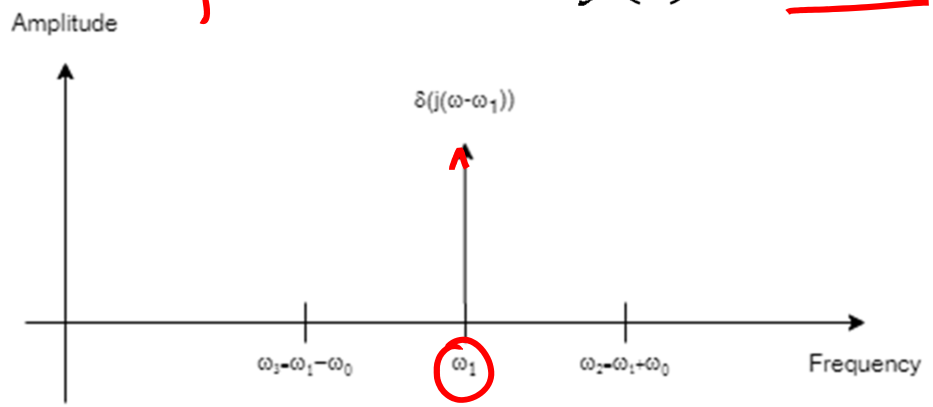
$$\boxed{X(j\omega) = \frac{1}{2}\delta\big(j(\omega - (\omega_1 - \omega_0))\big) + \frac{1}{2}\delta\big(j(\omega - (\omega_1 + \omega_0))\big)}$$

$\downarrow \omega_0$ $\uparrow \omega_0$

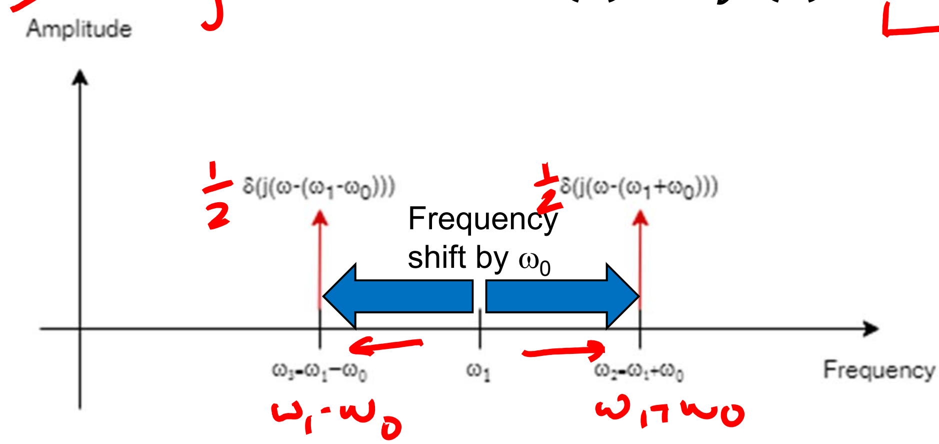
Multiplication by a real sinusoid in the time domain creates two signals at sum and difference frequencies

Graphically

$\cancel{X(j\omega)}$ $Y(j\omega)$ $$y(t) = \underline{e^{-j\omega_1 t}}$$



$\cancel{Y(j\omega)}$ $X(j\omega)$ $$x(t) = y(t) * \underline{\cos(\omega_0 t)}$$



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Also note the trigonometric identity

$$x(t) = \cos(\omega_0 t) * \cos(\omega_1 t) \qquad \cos(\omega_0 t) = \frac{1}{2}\left(e^{j\omega_0 t} + e^{-j\omega_0 t}\right)$$

$$x(t) = \frac{1}{4}\left(e^{j\omega_0 t} + e^{-j\omega_0 t}\right) * \left(e^{j\omega_1 t} + e^{-j\omega_1 t}\right)$$

$-\omega_0 + \omega_1 = -(\omega_0 - \omega_1)$

$$x(t) = \frac{1}{4}\left(e^{j(\omega_0-\omega_1)t} + e^{-j(\omega_0-\omega_1)t}\right) + \frac{1}{4}\left(e^{j(\omega_0+\omega_1)t} + e^{-j(\omega_0+\omega_1)t}\right)$$

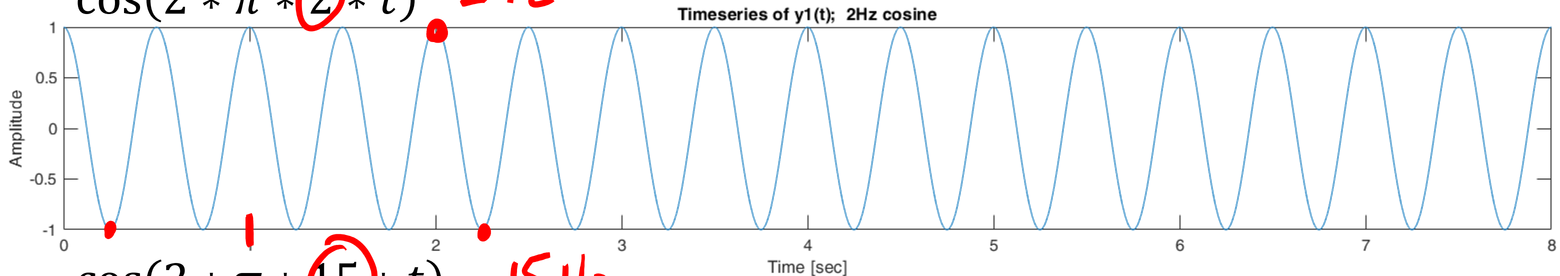
$$x(t) = \frac{1}{2}\cos\left((\omega_0 + \omega_1)t\right) + \frac{1}{2}\cos\left((\omega_0 - \omega_1)t\right)$$

The result of multiplying two sinusoids together results in two new sinsusoids at sum and difference frequencies.

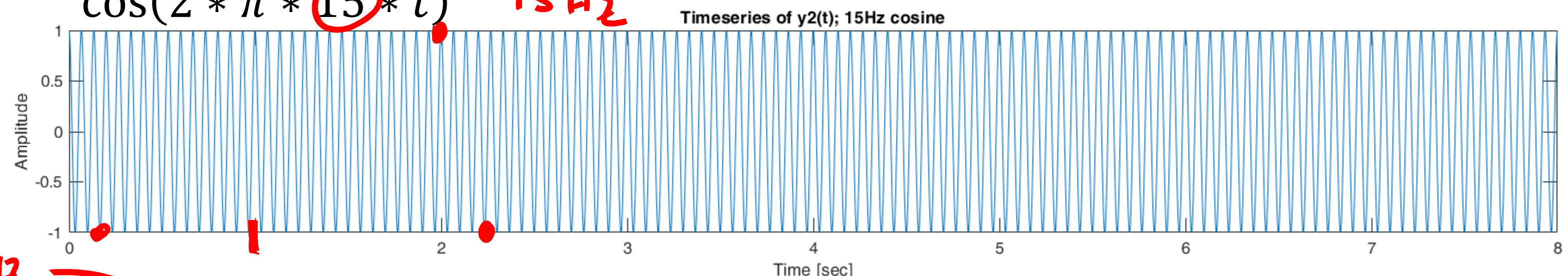
Sum Example : Time series

$$x(t) = \cos(\omega_0 t) + \cos(\omega_1 t)$$

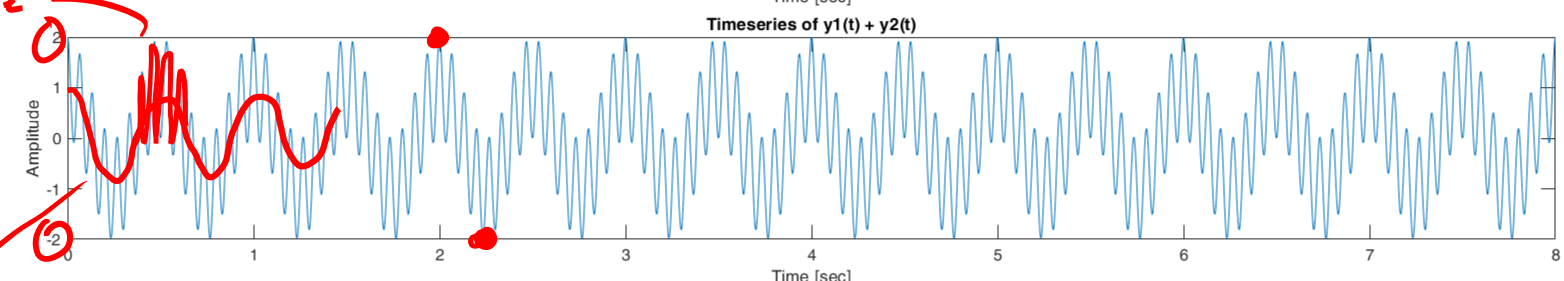
$\cos(2 * \pi * 2 * t)$ 2Hz



$\cos(2 * \pi * 15 * t)$ 15Hz



15Hz
2



2Hz (-2

$\cos(2 * \pi * 2 * t) + \cos(2 * \pi * 15 * t)$

Sum Example : Spectrum

$y_1(t)$ $y_2(t)$

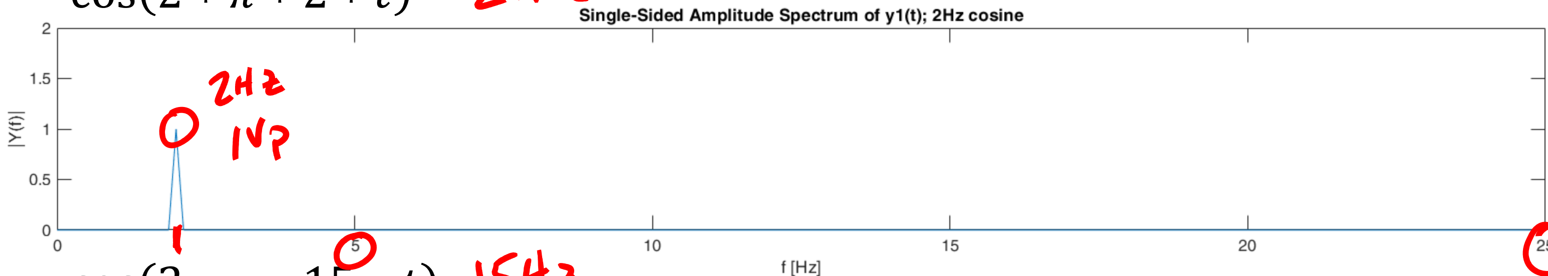
$$x(t) = \cos(\omega_0 t) + \cos(\omega_1 t)$$

FFT

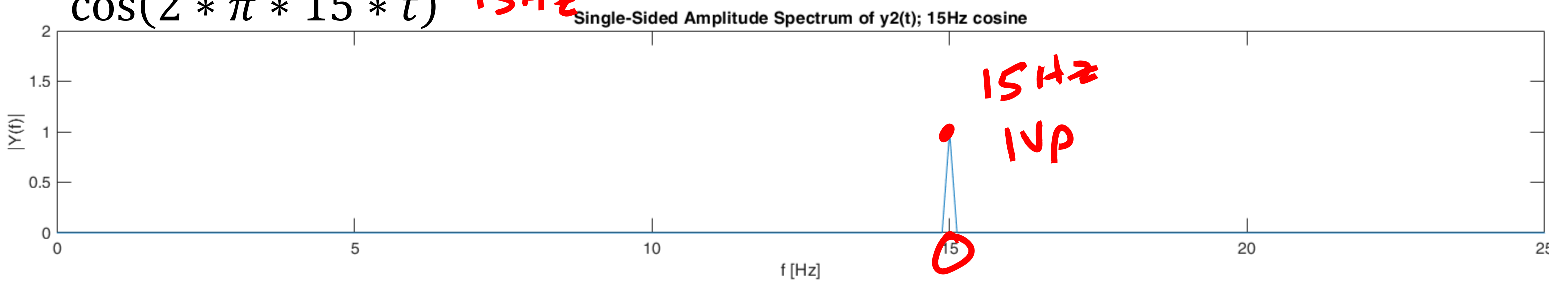
$X(j\omega) = Y_1(j\omega) + Y_2(j\omega)$

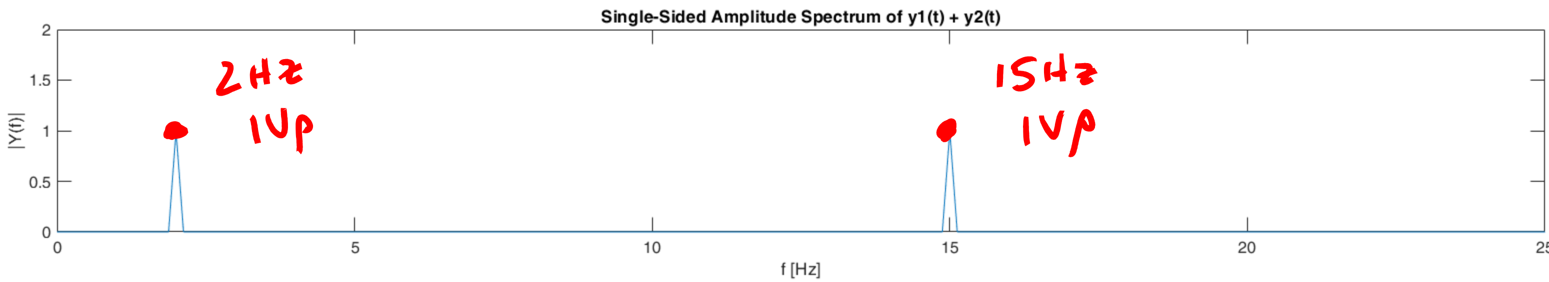
$\cos(2 * \pi * 2 * t)$ 2Hz

Mag Amp



$\cos(2 * \pi * 15 * t)$ 15Hz



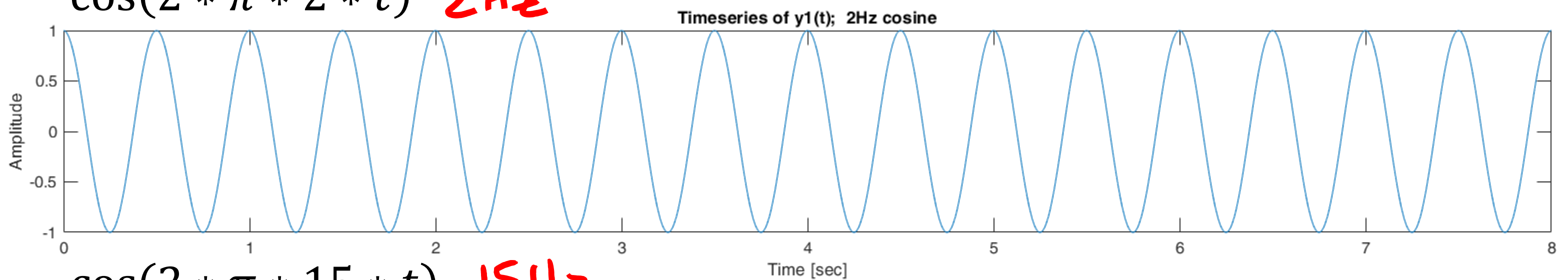


$$\left(\underline{\cos(2 * \pi * 2 * t) + \cos(2 * \pi * 15 * t)}\right)$$

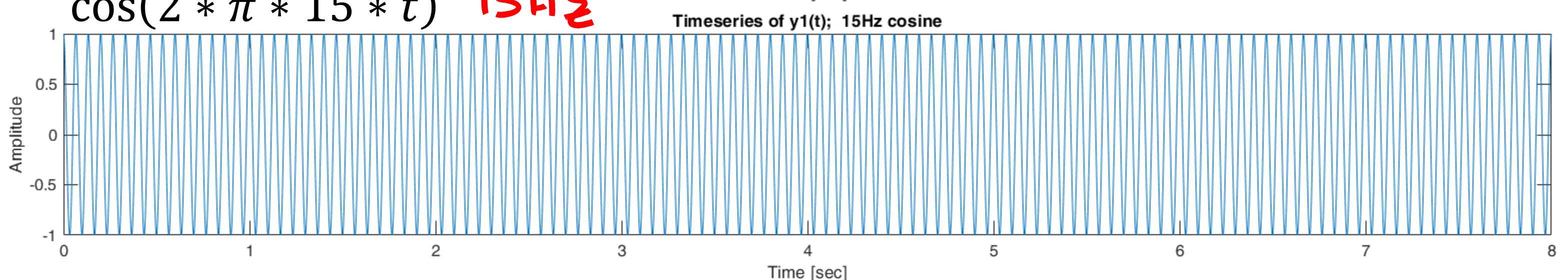
Product Example : Time series

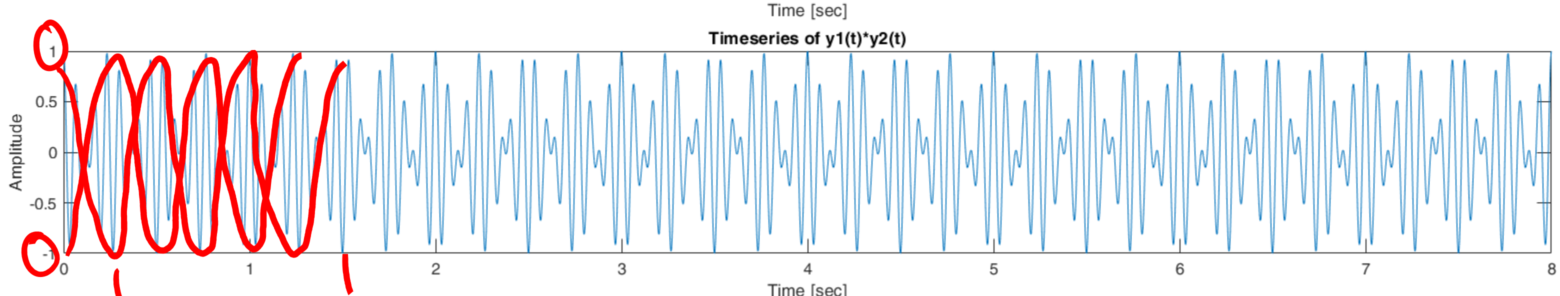
$$x(t) = \underline{\cos(\omega_0 t)} * \underline{\cos(\omega_1 t)}$$

$\cos(2 * \pi * 2 * t)$ 2Hz



$\cos(2 * \pi * 15 * t)$ 15Hz





2Hz 15Hz

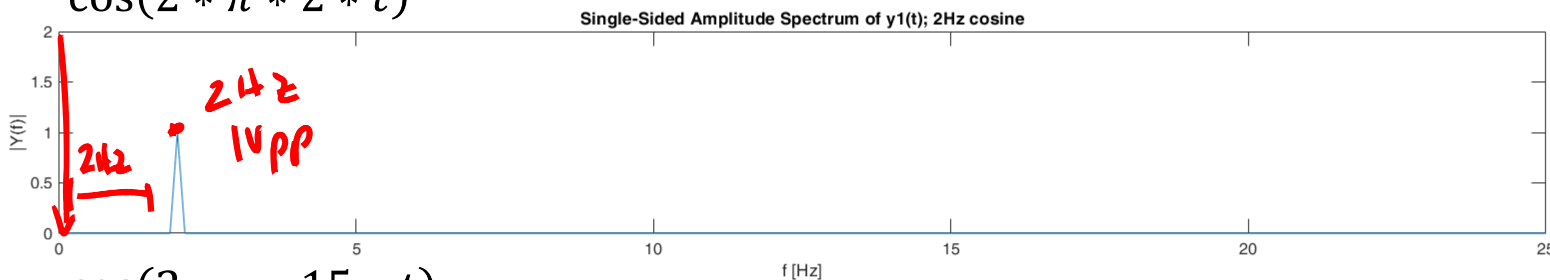
$$\underbrace{\cos(2 * \pi * 2 * t) * \cos(2 * \pi * 15 * t)}$$

Product Example : Spectrum

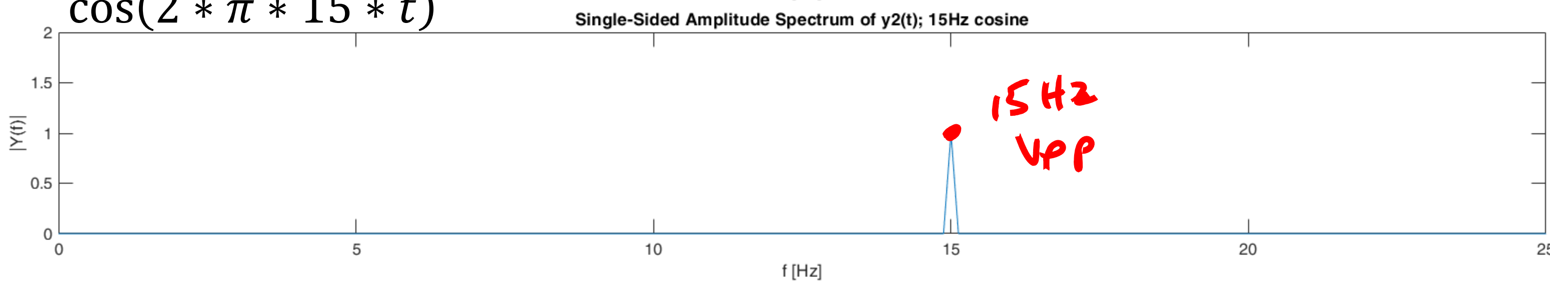
$$x(t) = \cos(\omega_0 t) * \cos(\omega_1 t) \qquad X(j\omega)$$

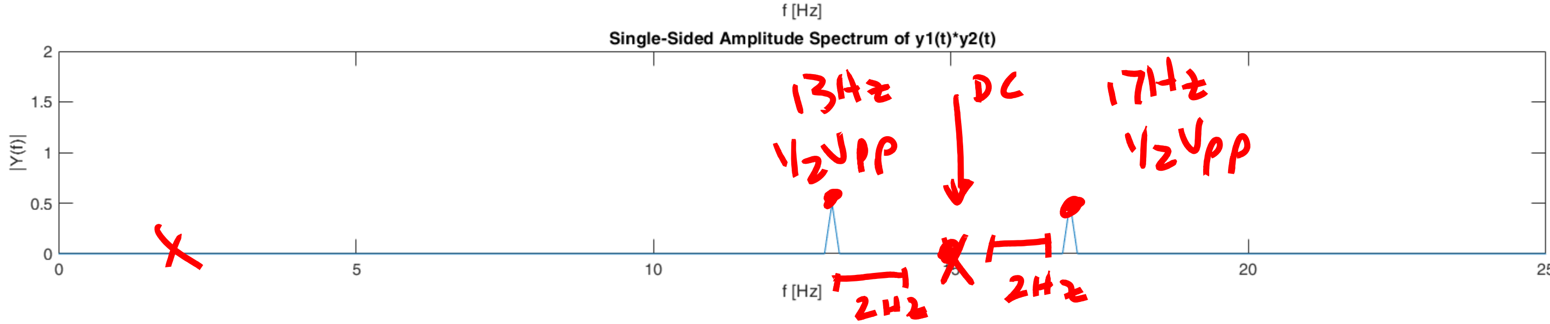
DC = 0Hz

$\cos(2 * \pi * 2 * t)$



$\cos(2 * \pi * 15 * t)$





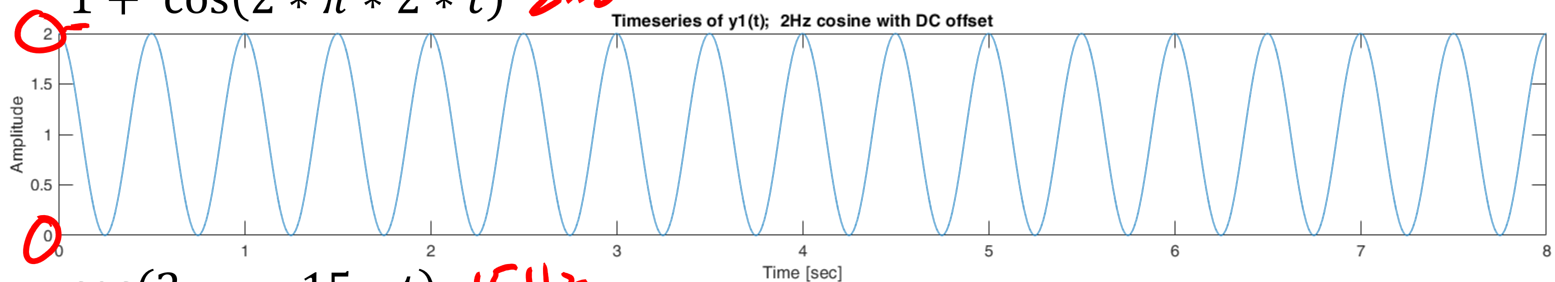
$$\underline{\cos(2 * \pi * 2 * t) * \cos(2 * \pi * 15 * t)}$$

$$0.5 * \cos(2 * \pi * 13 * t) + 0.5 * \cos(2 * \pi * 17 * t)$$

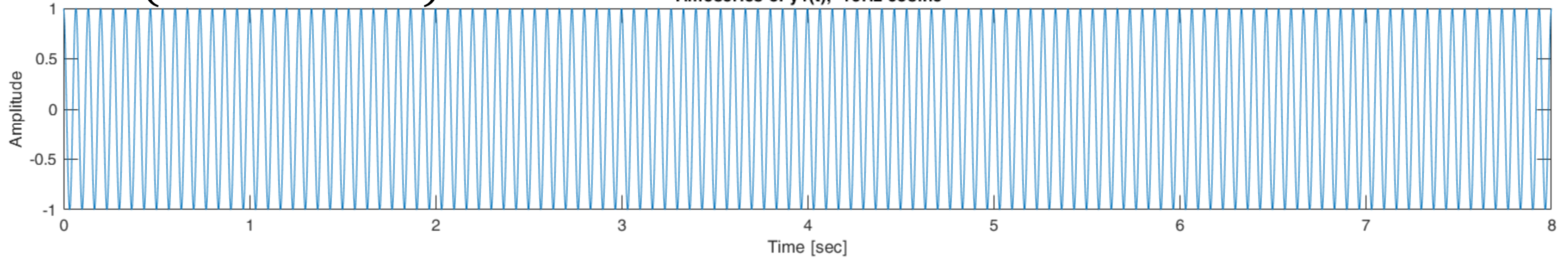
Product Example : Time series

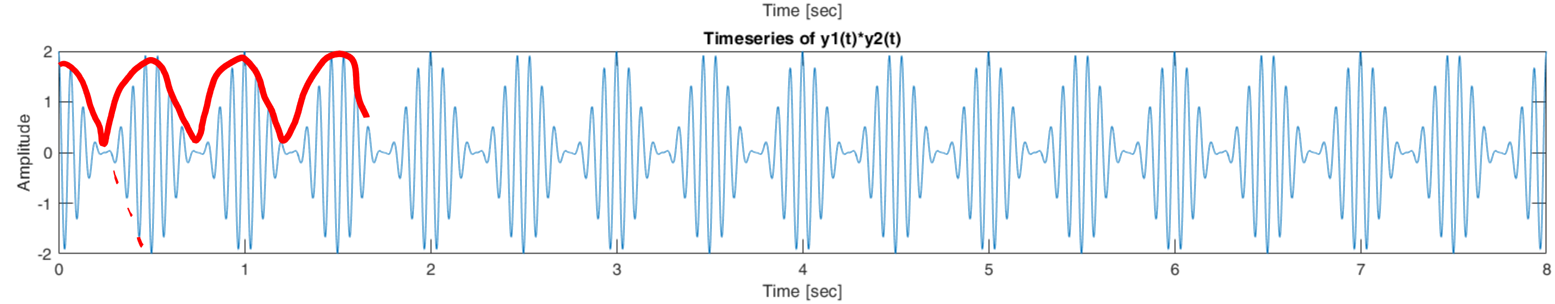
$$x(t) = \overbrace{(1 + \cos(\omega_0 t))}^{A(t)\,*} * \cos(\omega_1 t) \; : A(t)\cos(\omega_1 t)$$

$1 + \cos(2*\pi*2*t)$ 2Hz



$\cos(2*\pi*15*t)$ 15Hz



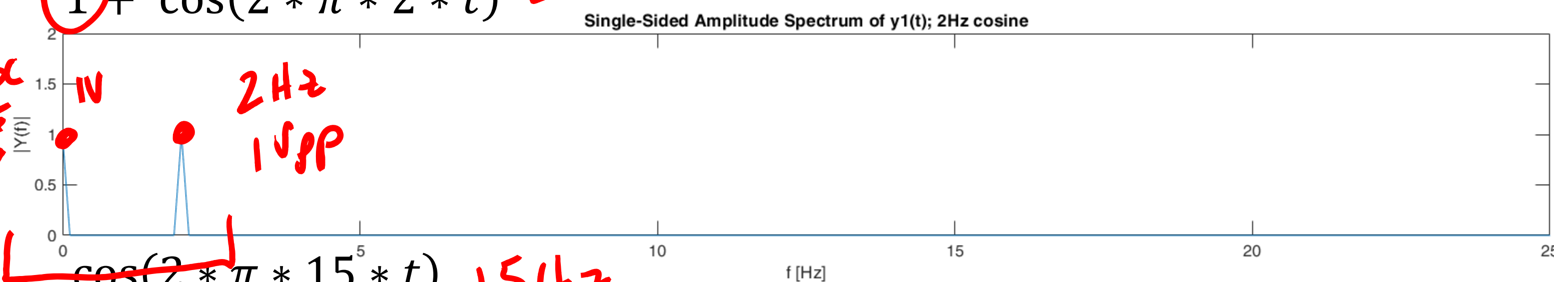


$(1 + \cos(2*\pi*2*t) +* \cos(2*\pi*15*t)$

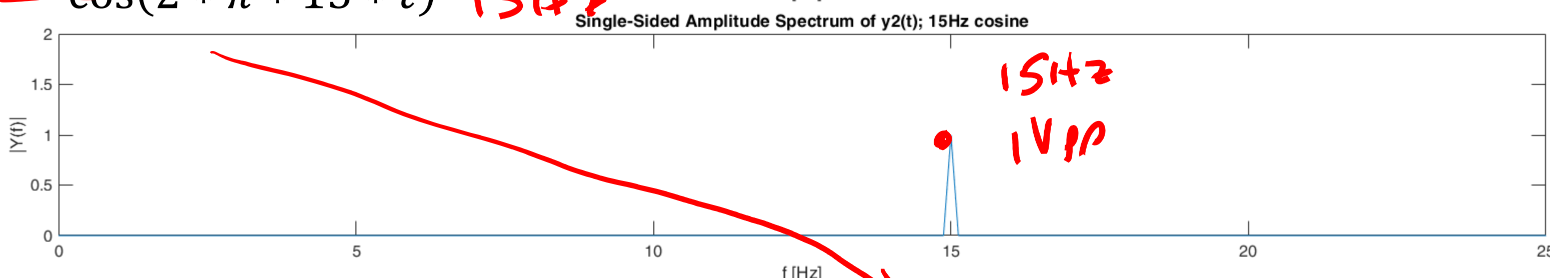
Product Example : Spectrum

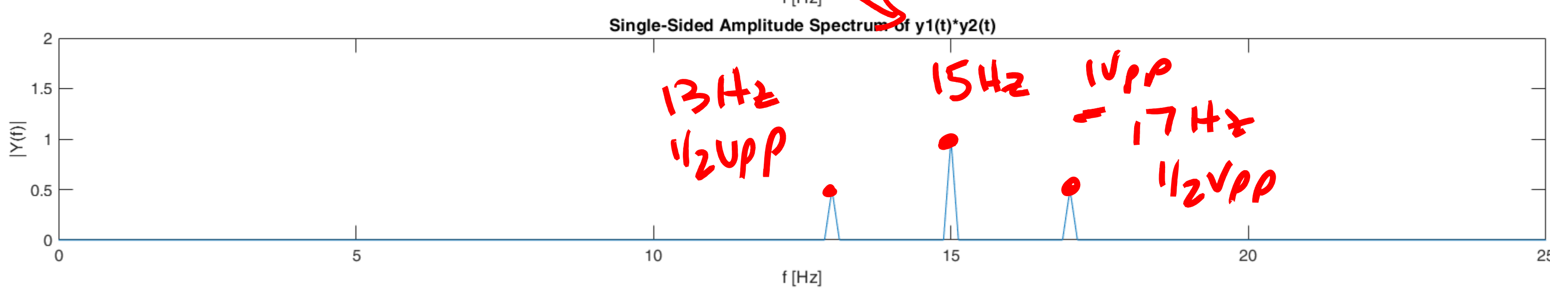
$$x(t) = (1 + \cos(\omega_0 t)) * \cos(\omega_1 t)$$

$1 + \cos(2 * \pi * 2 * t)$ 2Hz



$\cos(2 * \pi * 15 * t)$ 15Hz



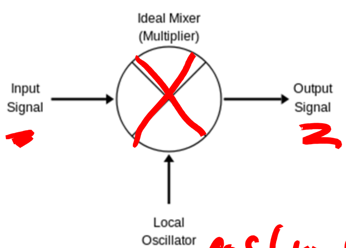


$$(1 + \cos(2 * \pi * 2 * t)) * \cos(2 * \pi * 15 * t)$$

$$0.5 * \cos(2 * \pi * 13 * t) + \cos(2 * \pi * 15 * t) + 0.5 * \cos(2 * \pi * 17 * t)$$

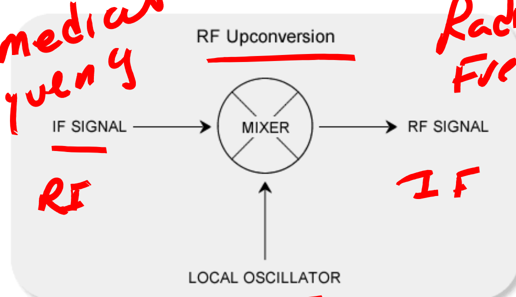
RF Mixer

- Used for frequency translation 1
  - To move modulated signal (carrier + information) to designated frequency band for transmission
  - To downconvert, moving modulated signal to lower intermediate frequency (IF) or baseband (0 Hz carrier) to extract the information signal



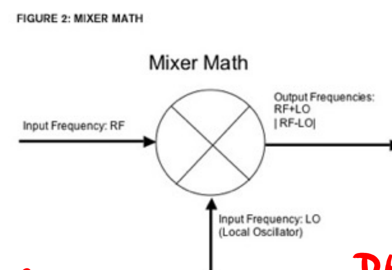
1 2 3 $\cos(\omega_0 t)$

Intermediate Frequency



Radio Frequency

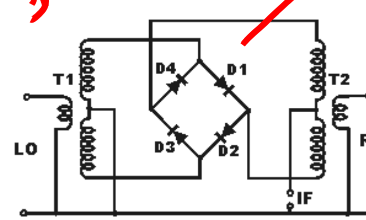
RF IF



Transformer

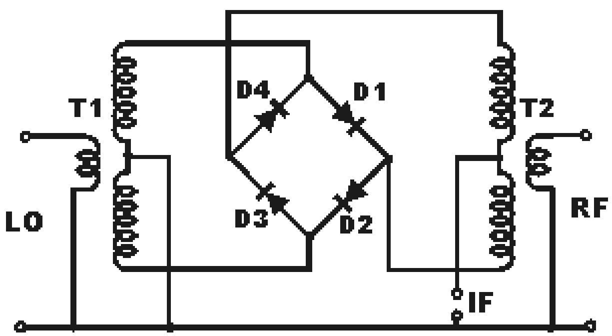
Bridge rect.

SMA

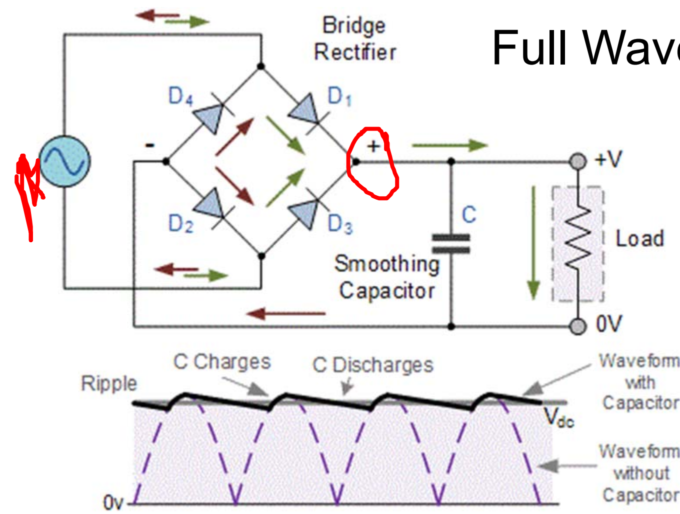


circuit schematic
double balanced mixer

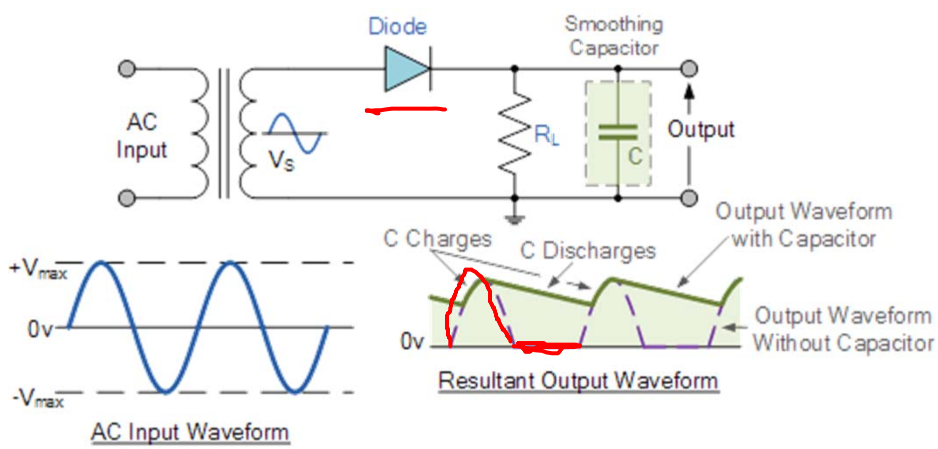
Recall the Diode Rectifier



Full Wave Rectifier



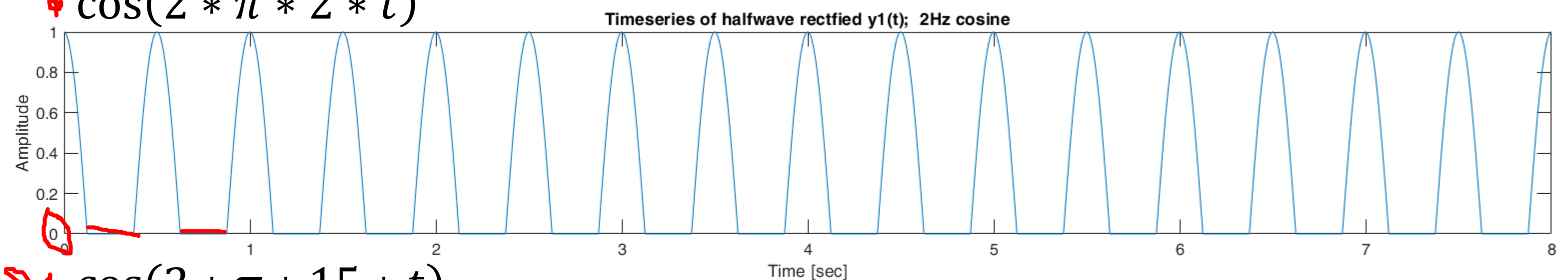
Half Wave Rectifier



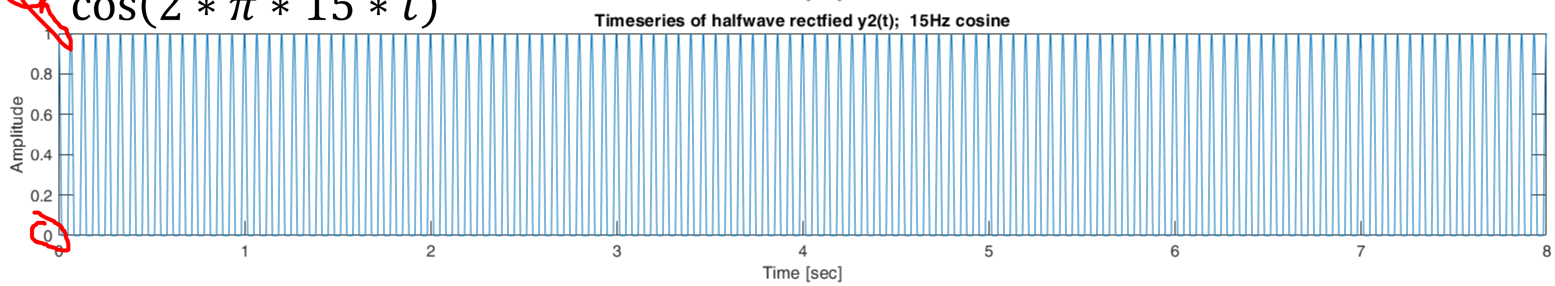
Sum Example Rectification: Time series

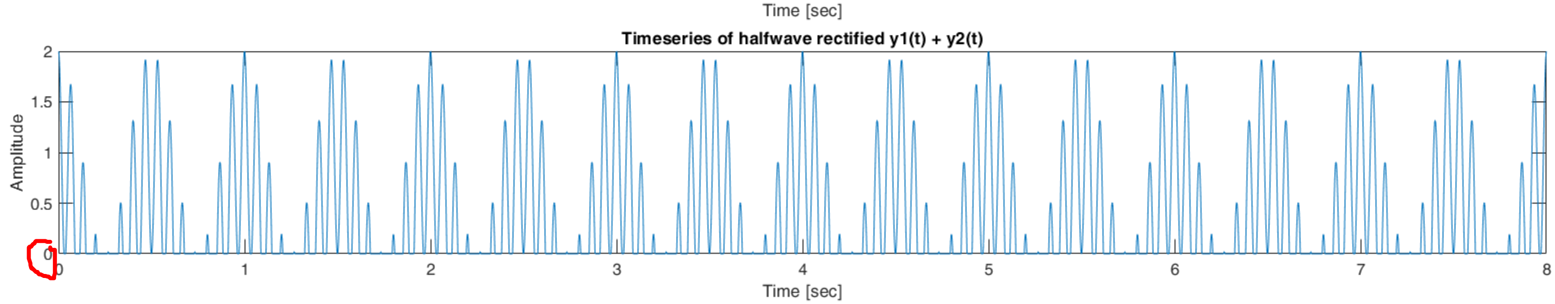
$$x(t) = \cos(\omega_0 t) + \cos(\omega_1 t)$$

$\cos(2 * \pi * 2 * t)$



$\cos(2 * \pi * 15 * t)$



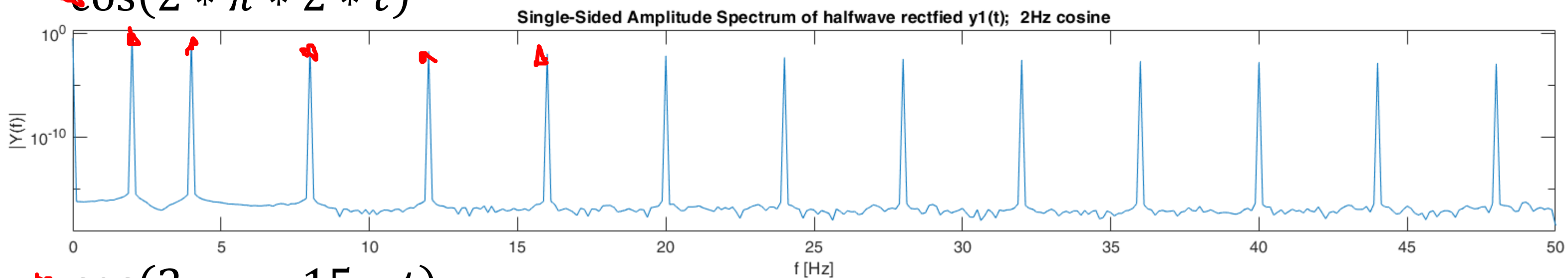


$\cos(2 * \pi * 2 * t) + \cos(2 * \pi * 15 * t)$

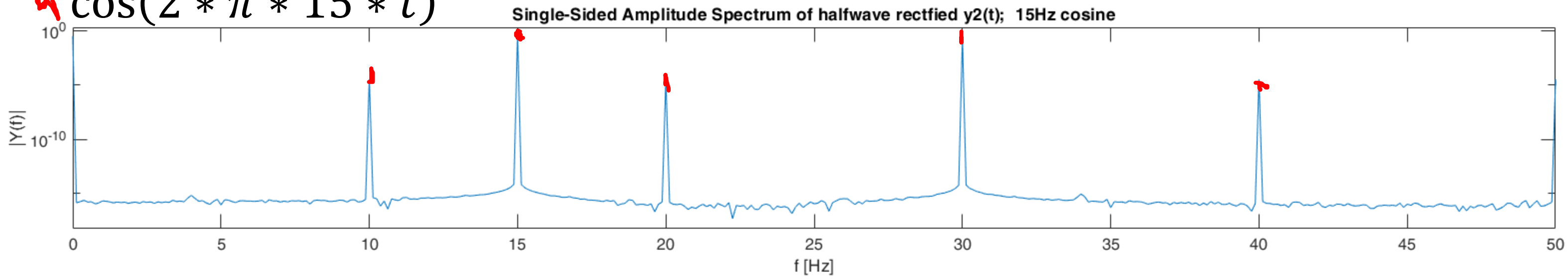
Sum Example Rectification : Spectrum

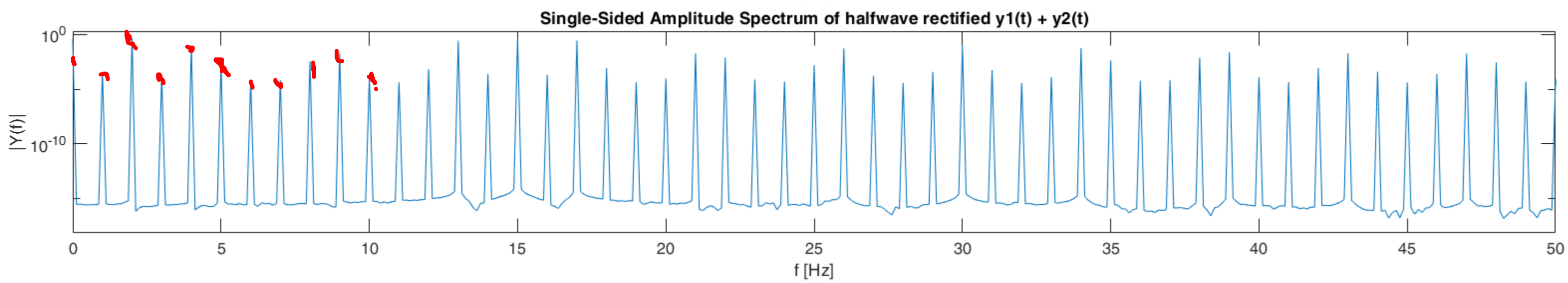
$$x(t) = \cos(\omega_0 t) + \cos(\omega_1 t)$$

$\cos(2 * \pi * 2 * t)$



$\cos(2 * \pi * 15 * t)$



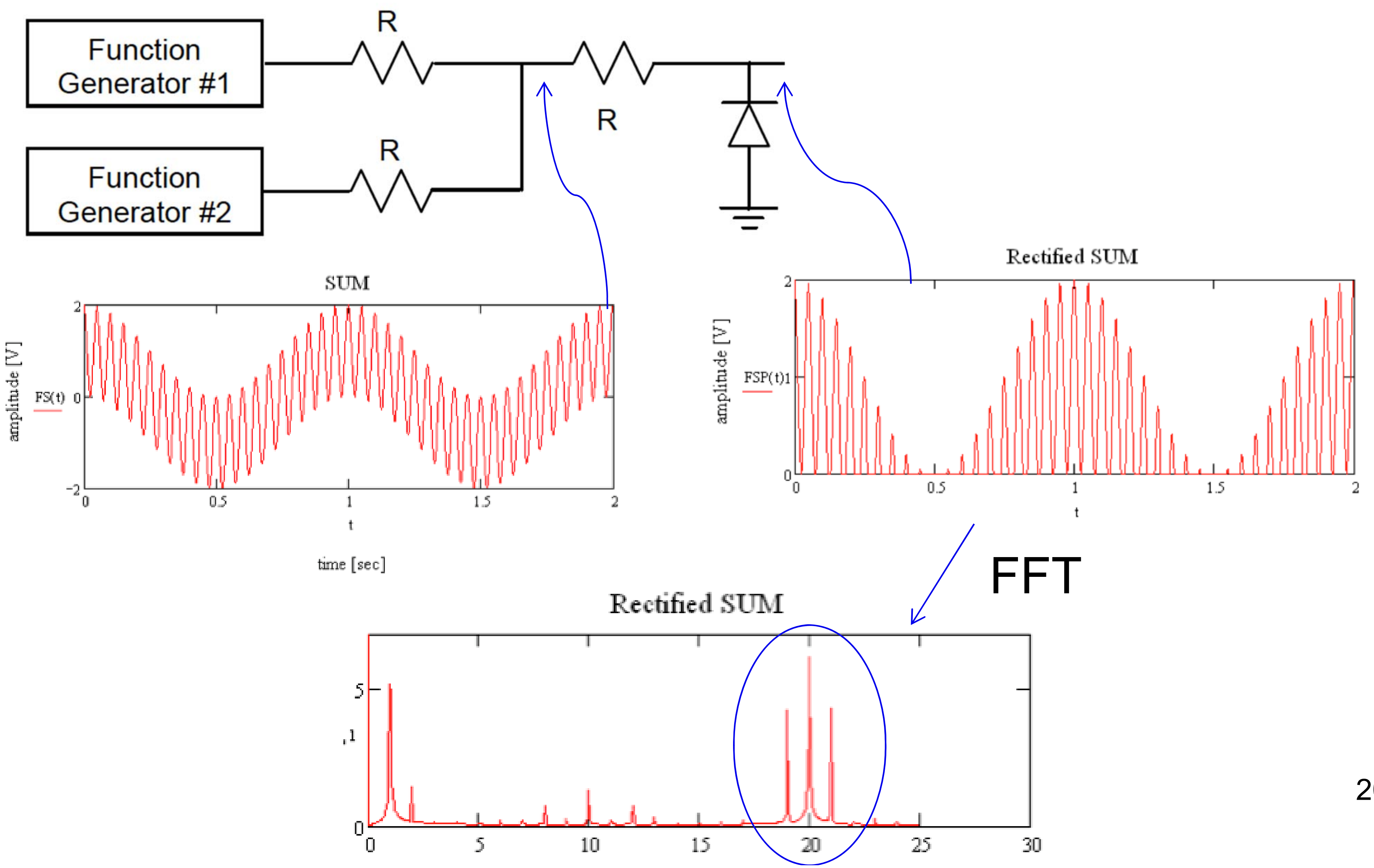


$\cos(2 * \pi * 2 * t) + \cos(2 * \pi * 15 * t)$

Circuit nonlinearities create complex harmonics!

Mixer Component of the Lab

- A diode rectifier can be used for simple mixing



Up-Conversion and Down-Conversion