Amplitude Modulation

Modulation

How can information be encoded on an EM wave?

$$A \cos(2\pi f_c t + \Phi)$$

Amplitude modulation: $A \rightarrow A(t)$
Frequency modulation: $f_c \rightarrow (f_c + f(t))$
Phase modulation: $\Phi \rightarrow \Phi(t)$

(*Recognize: Carrier frequency: f_c*)

What we want to know:

- the details of the modulation/demodulation,
- frequency spectrum,
- bandwidth of the modulated signal

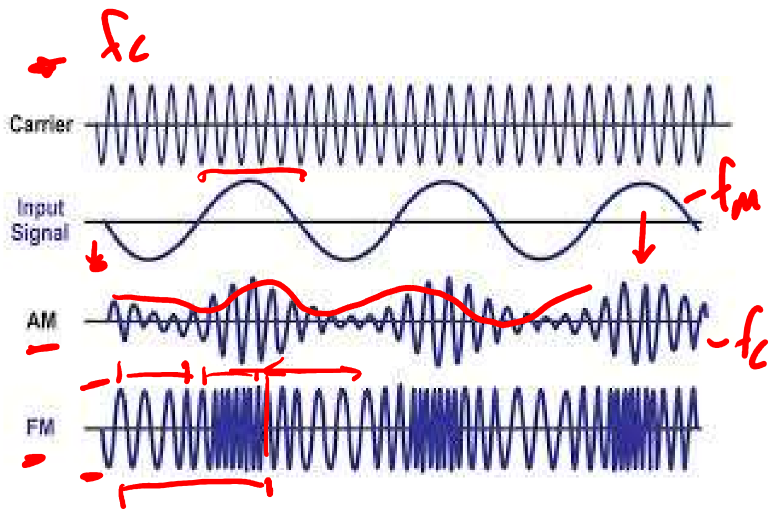
Note: We will only cover the coding of analog information (e.g. traditional AM radio), but there are other types of of encodings, for example digital (BPSK, QPSK, etc).

Modulation

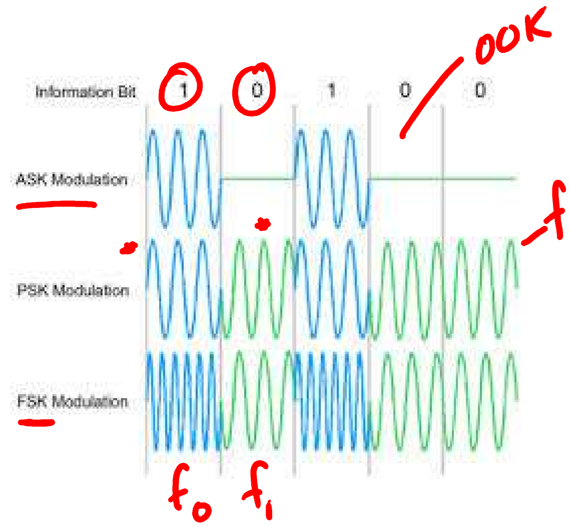
Information is transferred in the changing characteristic of the signal

Can change the amplitude, frequency or phase of the signal

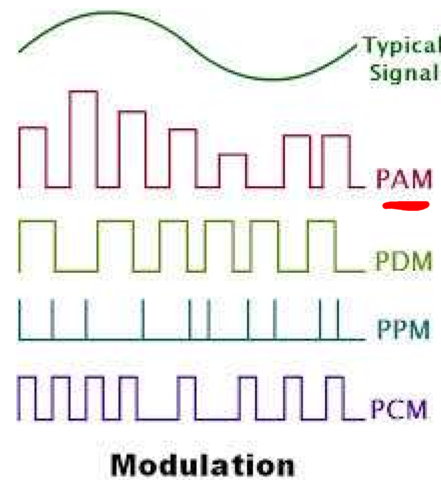
Modulation is needed for both wired and wireless communications



Analog Modulation



Digital Modulation



Pulse Modulation

Amplitude modulation

$x(t)$ – is the information we need to transmit

- Normalized between -1 and 1

AM radio has a defined limited bandwidth for $x(t)$: 40 Hz – 10.2 kHz

- m is the 'modulation depth'

f_c - carrier

$$\text{Am Signal} \quad S(t) = \underbrace{B}_{\text{const}} \overbrace{[1+\underbrace{m}_{\text{modulation Depth}}\, x(t)]}^{A(t)} \underbrace{\cos(2\pi f_c t)}_{f_c\text{- carrier frequency}}$$

Amplitude modulation

$$S(t) = B\,[\underbrace{1+m\,x(t)}_{A(t)}]\cos(2\pi f_c t)$$

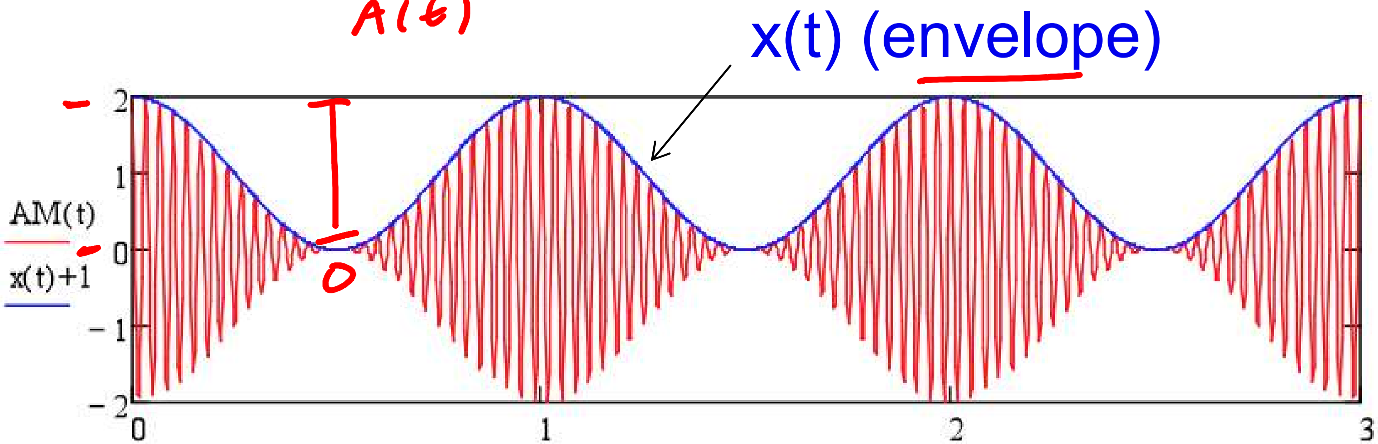
$x(t)=\cos(2\pi f_m t)$

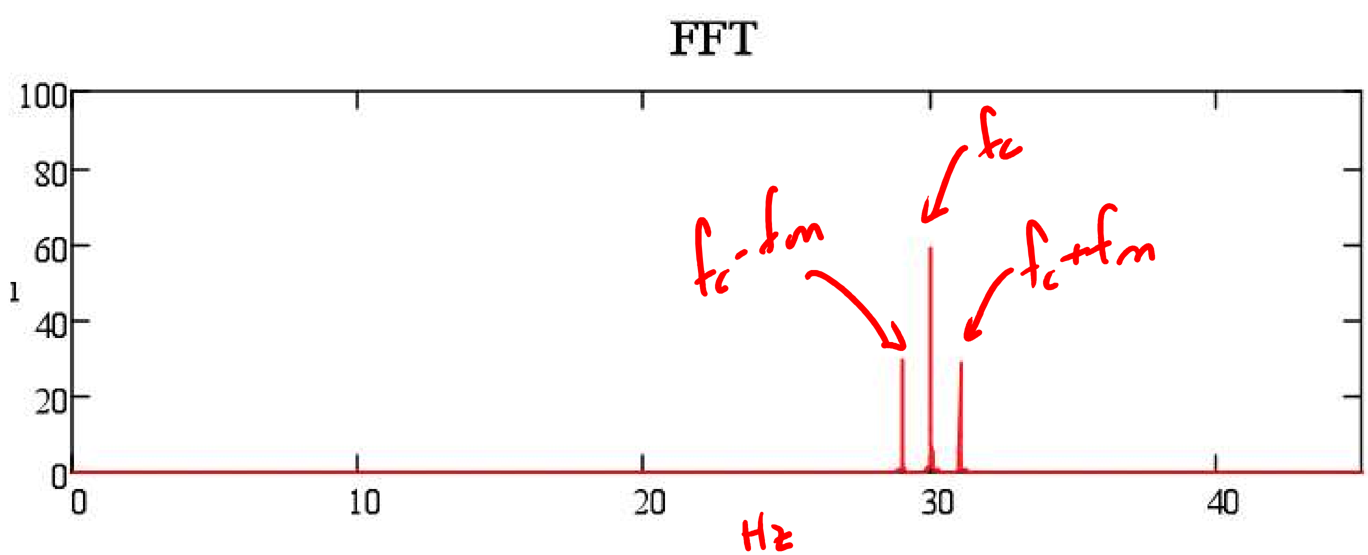
B = 1

f_c = 30

f_m =1

m = 1





ratio of amplitudes ~ modulation Depth

m = 0

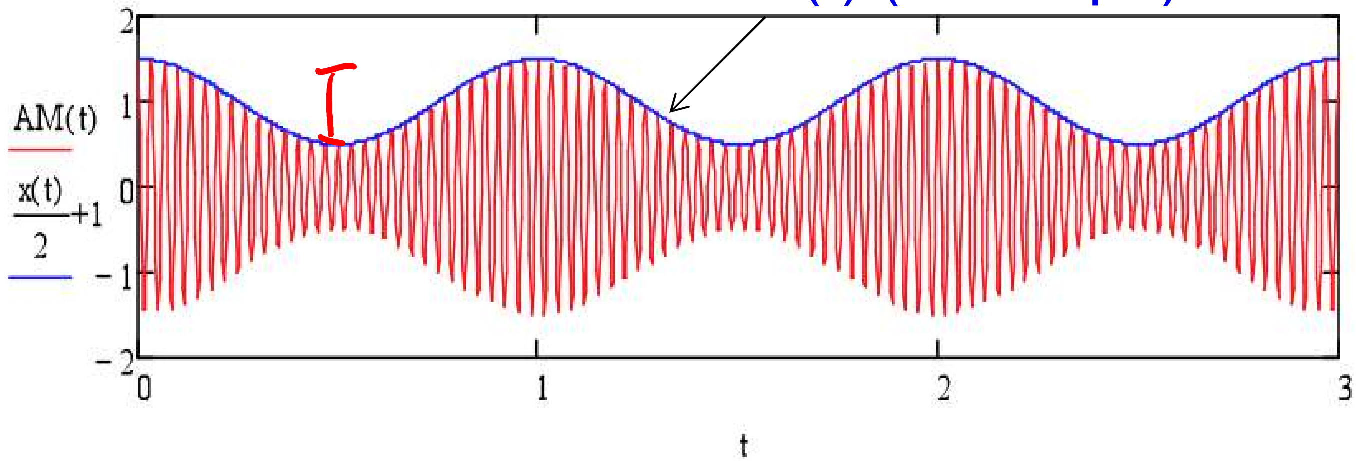
Amplitude modulation

0.5 -1

$S(t) = B\,[1+m\,x(t)]\,\cos(2\pi f_c t)$

1+.5 = 1.5
1-0.5=0.5

x(t) (envelope)



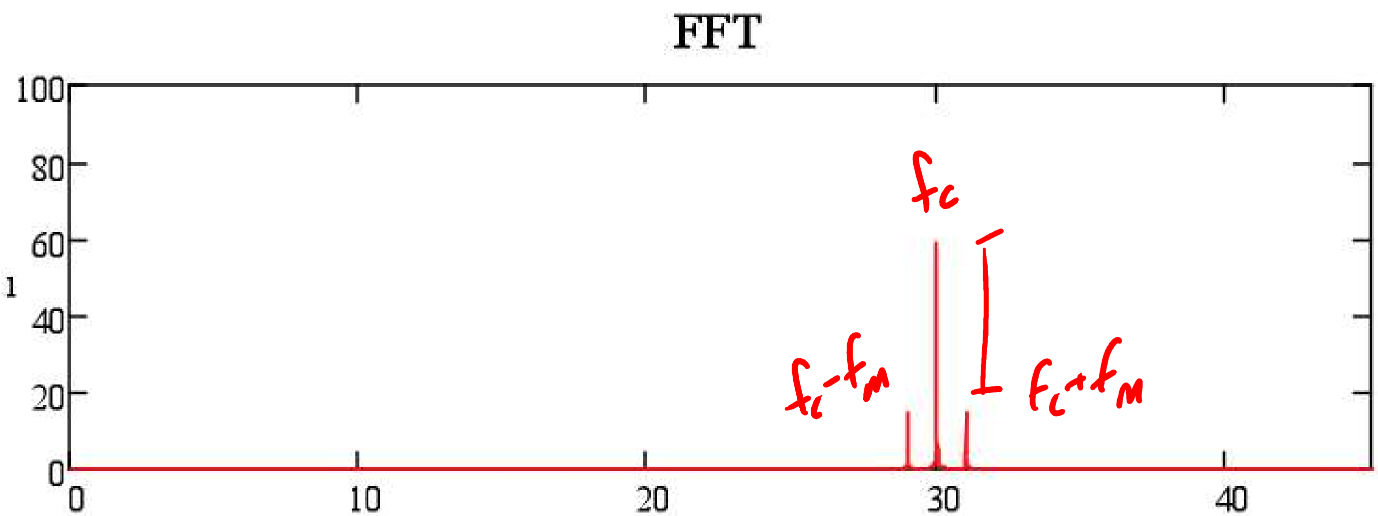
$x(t)=\cos(2\pi f_m t)$

$B = 1$

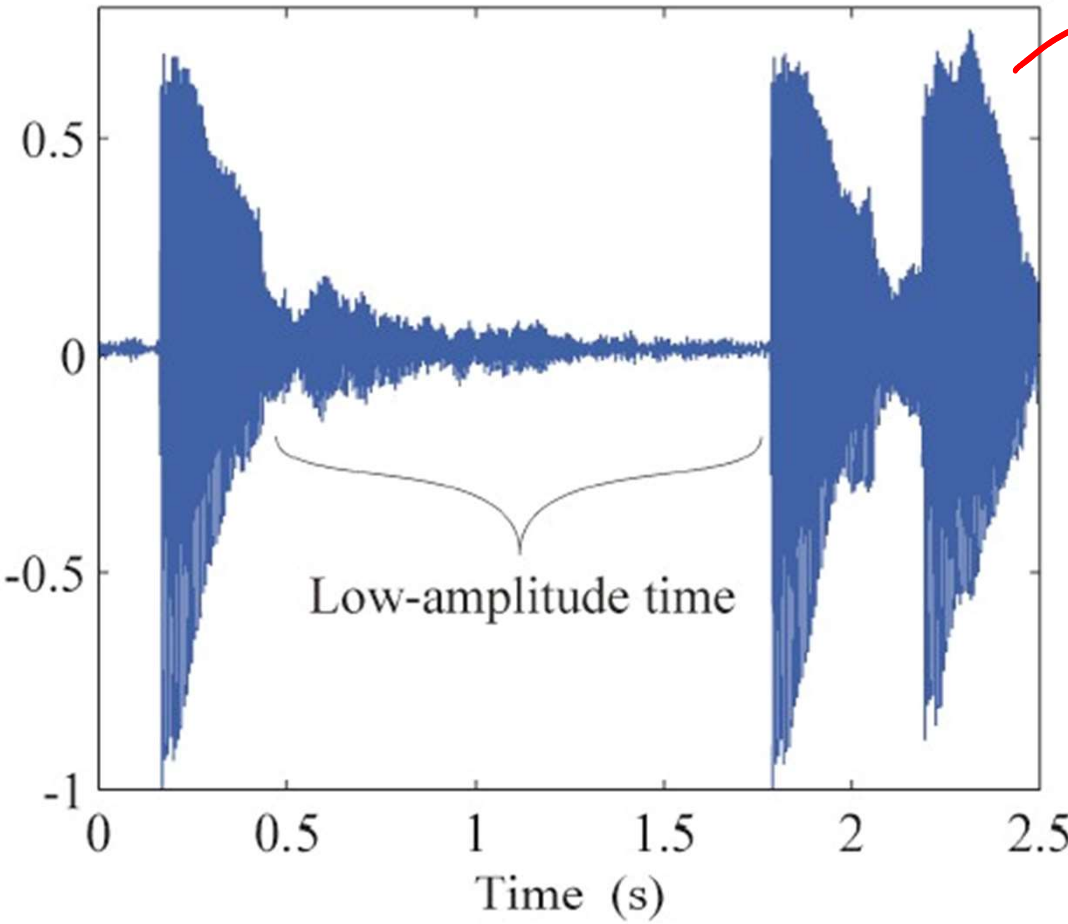
$f_c = 30$

$f_m = 1$

$m = 0.5$



Music signal example

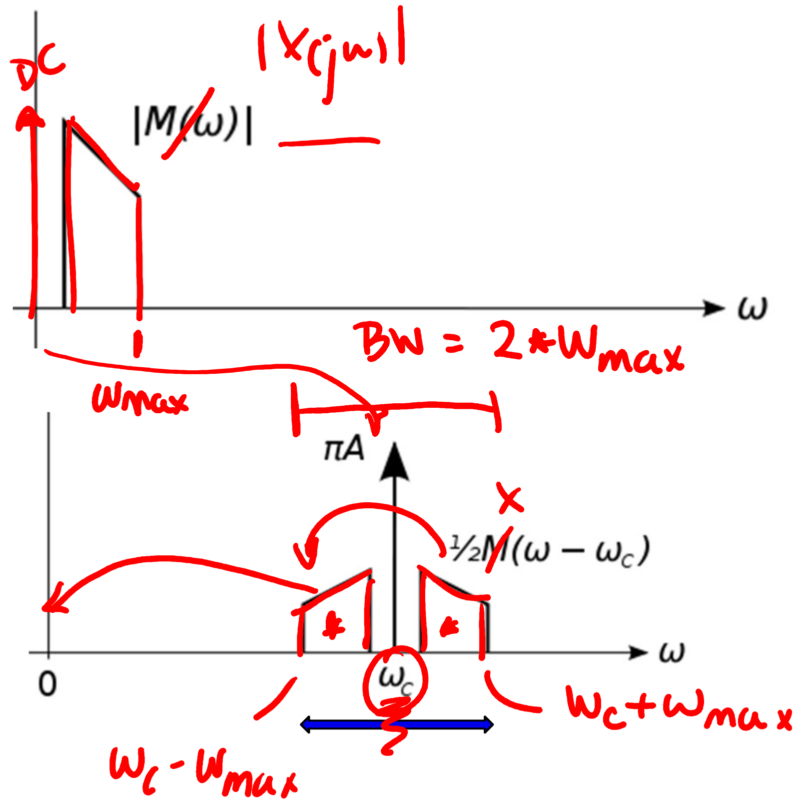


x(t)

Be aware that music or speech signals are highly variable with time.

Frequency spectrum of AM

$S(t) = B\,[1+m\,x(t)]\cos(2\pi f_c t)$



The frequency spectrum of x(t)

The frequency spectrum of S(t)

$w_c \gg w_{max}$

The bandwidth of S(t) is thus 2 x f_{max}, where f_{max} is the largest frequency present in signal x(t).

AM radio, f_{max} ~ 10 kHz, thus radio stations can be spaced ~20 kHz apart

AM demodulation

$S(t) = B\,[1+m\,\underbrace{\cos(2\pi f_m t)}_{x(t)}] * \cos(2\pi f_c t)$ – Am signal

$f_m \ll f_c$

$= B\,[\cos(2\pi f_c t)$

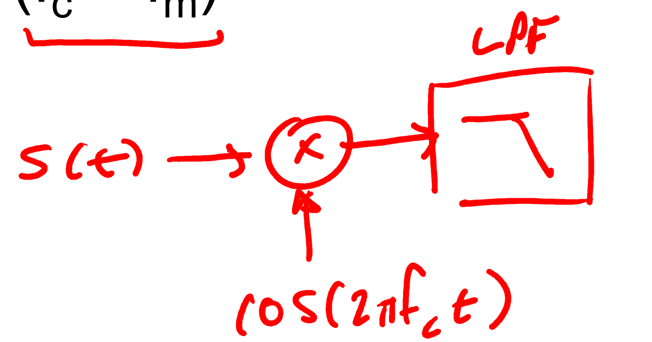
$+0.5m * \cos(2\pi (f_c+f_m)t) + 0.5m * \cos(2\pi (f_c-f_m)t)]$

- Frequency content of S(t): $(f_c - f_m)$, f_c, $(f_c + f_m)$

Demodulation

How to demodulate

Mix S(t) with $\cos(2\pi f_c t)$



$S_{demod}(t) = S(t) * \cos(2\pi f_c t)$

$= B\,[\cos(2\pi f_c t) *$

$+0.5m * \cos(2\pi (f_c+f_m)t) * + 0.5m * \cos(2\pi (f_c-f_m)t)] * \cos(2\pi f_c t)$

$= B\,[\cos(2\pi f_c t) * \cos(2\pi f_c t)$

$+ 0.5m * \cos(2\pi (f_c+f_m)t) * \cos(2\pi f_c t)$

$+ 0.5m * \cos(2\pi (f_c-f_m)t) * \cos(2\pi f_c t)]$

AM demodulation

$f_c+f_c=2f_c \quad f_c-f_c=0 \Leftarrow DC$

$f_c+f_m+f_c=2f_c+f_m$

$f_c+f_m-f_c=f_m$

$f_c-f_m+f_c=2f_c-f_m$

$=-f_m \quad f_c \gg f_m$

$S_{demod}(t) = B\,[\cos(2\pi f_c t)*\cos(2\pi f_c t)$

$+ 0.5m*\cos(2\pi (f_c+f_m)t)*\cos(2\pi f_c t)$

$+ 0.5m*\cos(2\pi (f_c-f_m)t)*\cos(2\pi f_c t)]$

$= B\,[0.5*\overbrace{\cos(2\pi\, DC\, t)}^{=1}$ + $0.5*\cos(2\pi\, 2*f_c t)$ $(DC = 0)$

$+ 0.25m*\cos(2\pi (f_m)t) + 0.25m*\cos(2\pi (2*f_c+f_m)t)$

$+ 0.25m*\cos(2\pi (-f_m)t) + 0.25m*\cos(2\pi (2*f_c-f_m)t)]$

$\cos(-x)=\cos(x) \quad \cos(2\pi(-f_m)) = \cos(2\pi f_m)$

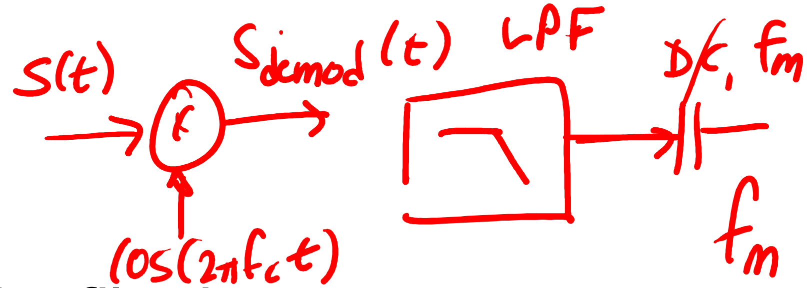
$= B\,[0.5*\overset{1}{\cancel{\cos(2\pi\, DC\, t)}}) + 0.5*\cos(2\pi\, 2*f_c t)$

$+ 0.5m*\cos(2\pi (f_m)t) + 0.25m*\cos(2\pi (2*f_c+f_m)t)$

$+ 0.25m*\cos(2\pi (2*f_c-f_m)t)]$

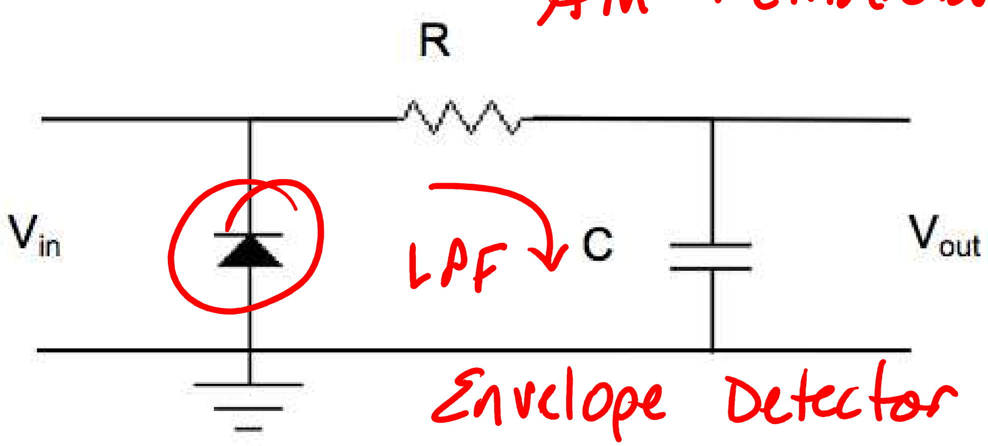
AM demodulation

$$S_{demod}(t) = B\,[0.5 + 0.5^*\cos(2\pi\, 2^*f_c t) + 0.5m^*\cos(2\pi\,(f_m)t) + 0.25m^*\cos(2\pi\,(2^*f_c+f_m)t) + 0.25m^*\cos(2\pi\,(2^*f_c-f_m)t)]$$



- DC, $2f_c$, $(2f_c - f_m)$, $(2f_c + f_m)$, f_m
- Remove unwanted frequencies by filtering
- Recover f_m

AM Demodulation



Envelope Detector

Diode: does the “mixing” (non-ideal)

RC low pass filter

AM demodulation

$S(t) = B\,[1+m\,x(t)]\cos(2\pi f_c t)$ - rectified

$x(t)=\cos(2\pi f_m t)$

$B = 1$

$f_c = 30$

$f_m = 1$

$m = 0.5$