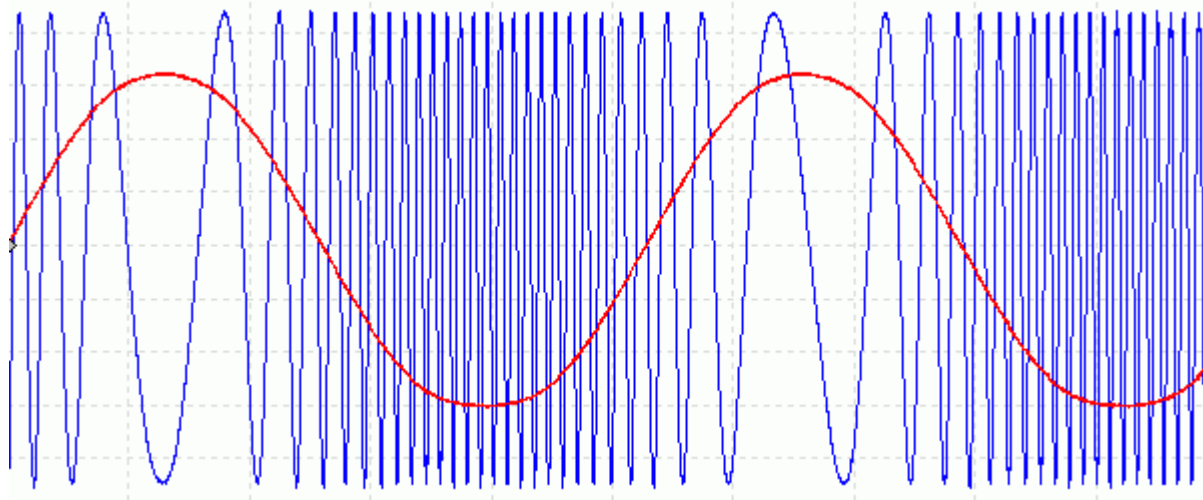
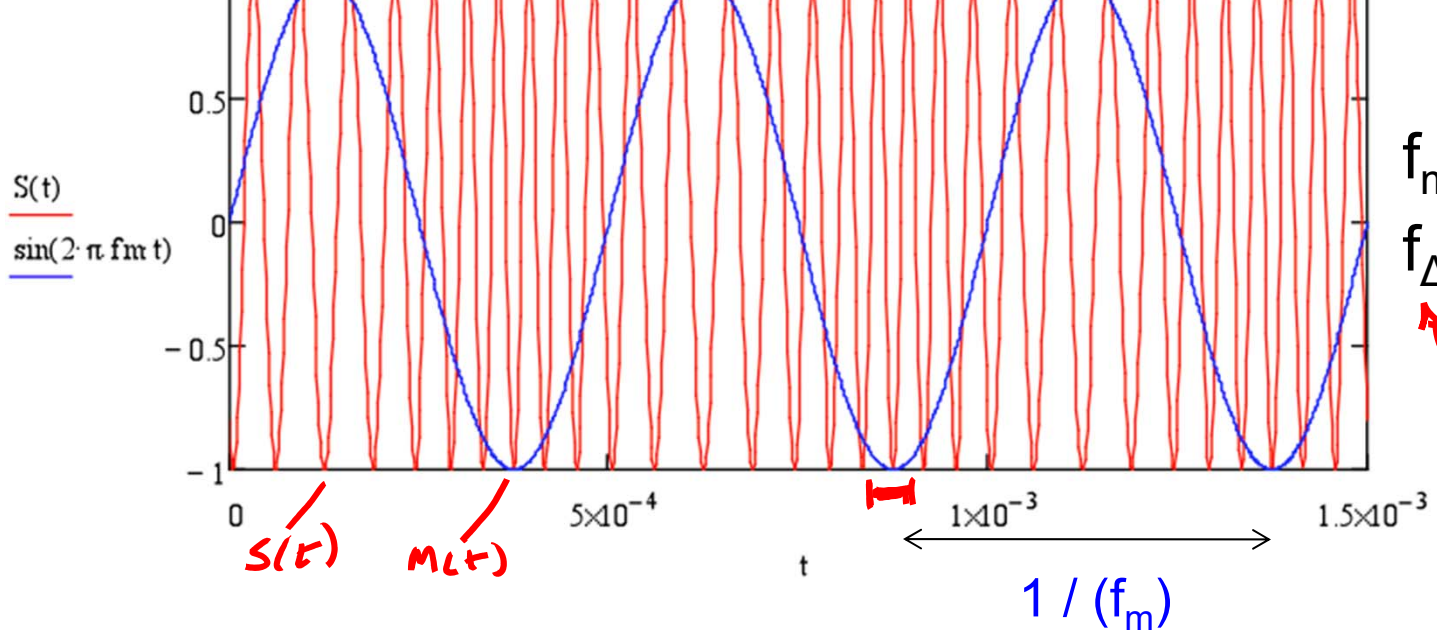


Frequency Modulation



Frequency/phase Modulation

- AM: $S(t) = B [1 + m \sin(2\pi f_m t)] \cos(2\pi f_c t)$
 $A(t) = 1 + m \sin(2\pi f_m t)$
 $\phi(t) = 2\pi f_c t$
 - FM: $S(t) = B \cos[2\pi f_c t + f_\Delta / f_m \cos(2\pi f_m t)]$
 $B \cdot A(t) \cdot \cos(\phi(t))$
 $A(t) = 1$
 $\phi(t) = 2\pi f_c t + \frac{f_\Delta}{f_m} \cos(2\pi f_m t)$
 $M(t)$
 $1 / (f_c + f_\Delta)$
 $1 / (f_c - f_\Delta)$
- $B = 1$ carrier
 $f_c = 20$ kHz
 $D = f_\Delta / f_m$



modulation frequency
 $f_m = 2$ kHz
 $f_\Delta = 5$ kHz
 frequency deviation

Frequency/phase Modulation

$$S(t) = B \cos[2\pi f_c t + f_{\Delta}/f_m \cos(2\pi f_m t)]$$

$$= B \cos[\Phi(t)]$$

$$f = \frac{1}{2\pi} \frac{d}{dt} \Phi(t)$$

$$m(t) = \cos(2\pi f_m t)$$

$$\Phi(t) = 2\pi f_0 t + \phi$$

$$\Rightarrow \Phi(t) = 2\pi f_c t + f_{\Delta}/f_m \cos(2\pi f_m t)$$

$$\frac{1}{2\pi} \frac{d}{dt} \Phi(t) = f_0$$

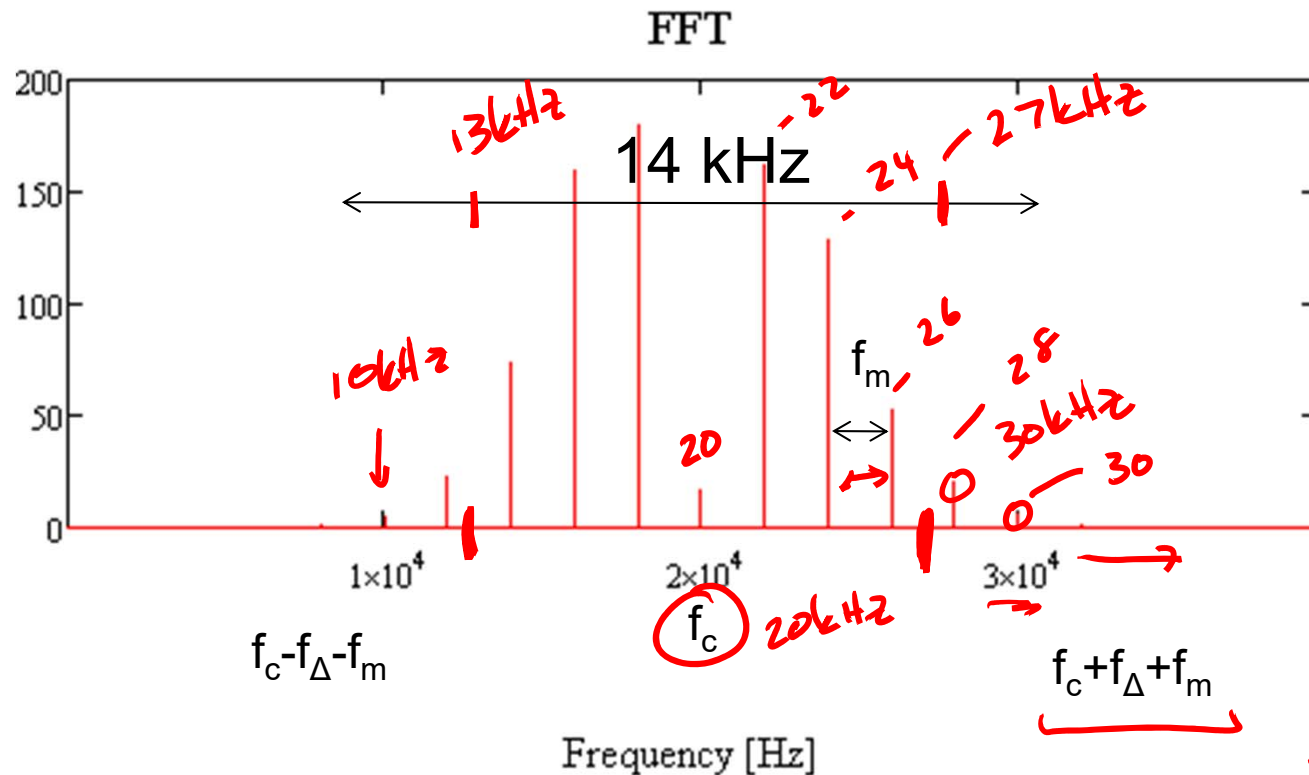
Instantaneous frequency:

$$f = \frac{d}{dt} \Phi(t) / (2\pi) = f_c + (f_m) * f_{\Delta} / f_m \sin(2\pi f_m t)$$

$f(t)$

$$f = f_c + f_{\Delta} \sin(2\pi f_m t); [f_c - f_{\Delta}; f_c + f_{\Delta}]$$

FM bandwidth



$$x(t) = \sin(2\pi f_m t)$$

$$B = 1$$

$$f_c = \underline{20 \text{ kHz}}$$

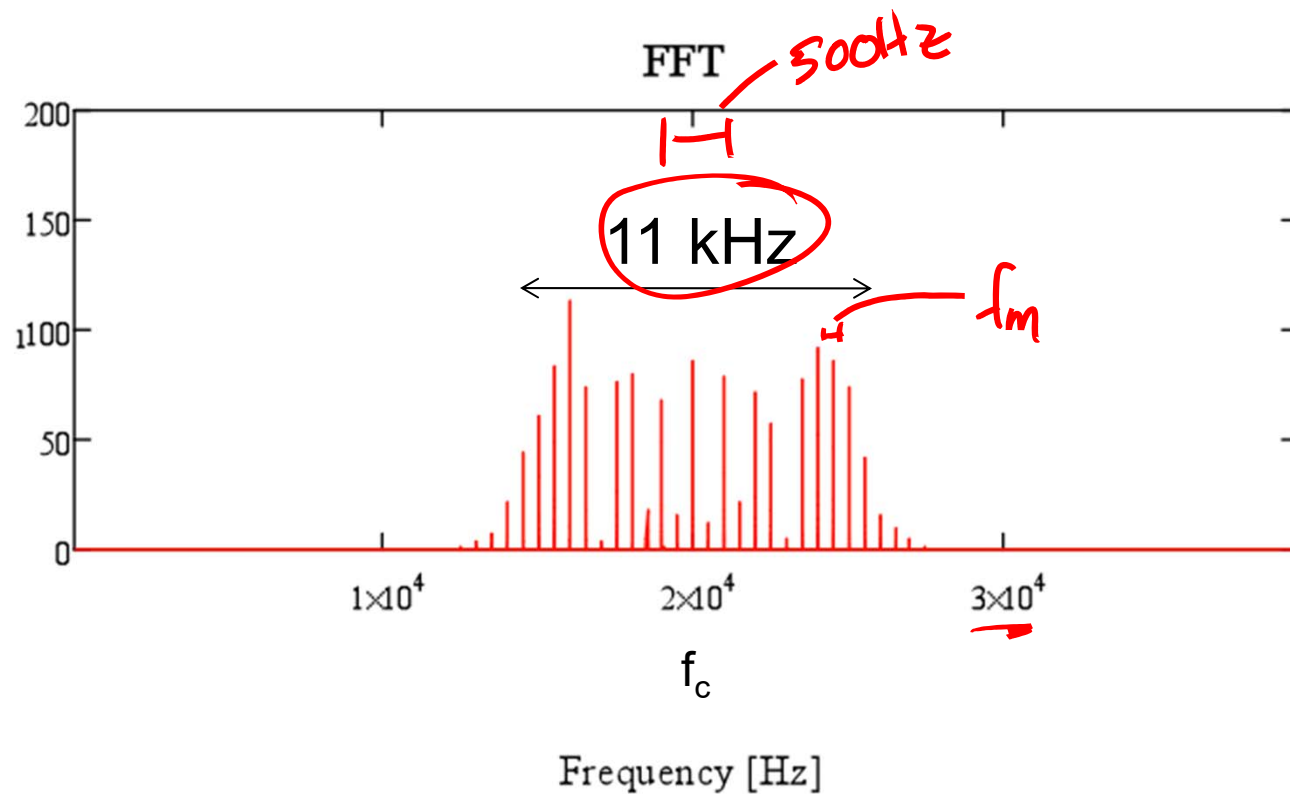
$$f_m = \underline{2 \text{ kHz}}$$

$$f_{\Delta} = \underline{5 \text{ kHz}}$$

$$\text{BW} \approx \underline{2(f_{\Delta} + f_m)} = 2(\underline{5 \text{ kHz}} + \underline{2 \text{ kHz}}) = \underline{14 \text{ kHz}}$$

(Carson rule – approximation of the BW (~98%) for FM signal)

FM bandwidth



$$x(t) = \sin(2\pi f_m t)$$

$$B = 1$$

$$f_c = \underline{20 \text{ kHz}}$$

$$f_m = \underline{500 \text{ Hz}}$$

$$\underline{f_\Delta = 5 \text{ kHz}}$$

AM signal
 $f_m = 500 \text{ Hz}$

$$BW \approx 2(f_\Delta + f_m) = 2(5\text{kHz} + 0.5\text{kHz}) = 11 \text{ kHz}$$

$$f_\Delta \gg f_m \quad BW \sim 2f_\Delta$$

FM Theory

$$S(t) = B \cos \left[2\pi f_c t + \frac{f_\Delta}{f_m} \cos(2\pi f_m t) \right]$$

$$S(t) = A J_0(x) \cos(2\pi f_c t)$$

$$+ A \sum_{n=1}^{\infty} J_{2n}(x) \sin[2\pi(f_c + 2nf_m)t]$$

$$+ A \sum_{n=1}^{\infty} J_{2n}(x) \sin[2\pi(f_c - 2nf_m)t]$$

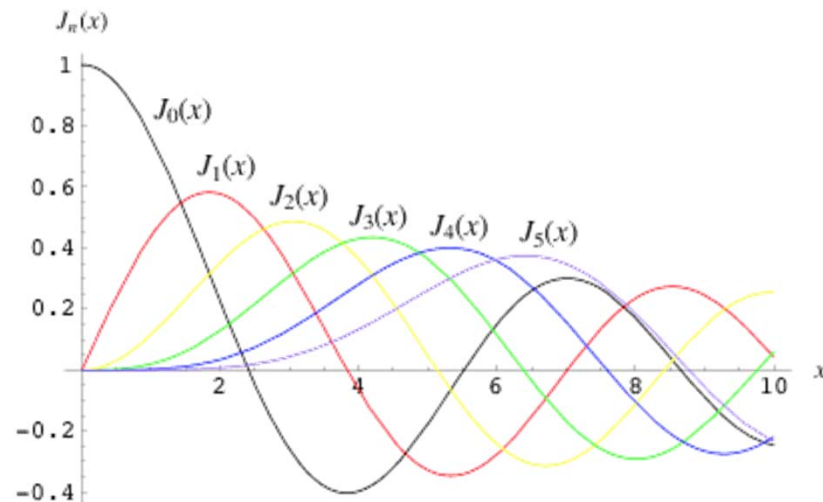
$$+ A \sum_{n=1}^{\infty} J_{2n-1}(x) \cos[2\pi(f_c + (2n-1)f_m)t]$$

$$+ A \sum_{n=1}^{\infty} J_{2n-1}(x) \cos[2\pi(f_c - (2n-1)f_m)t]$$

even multiples of f_m Fourier Series

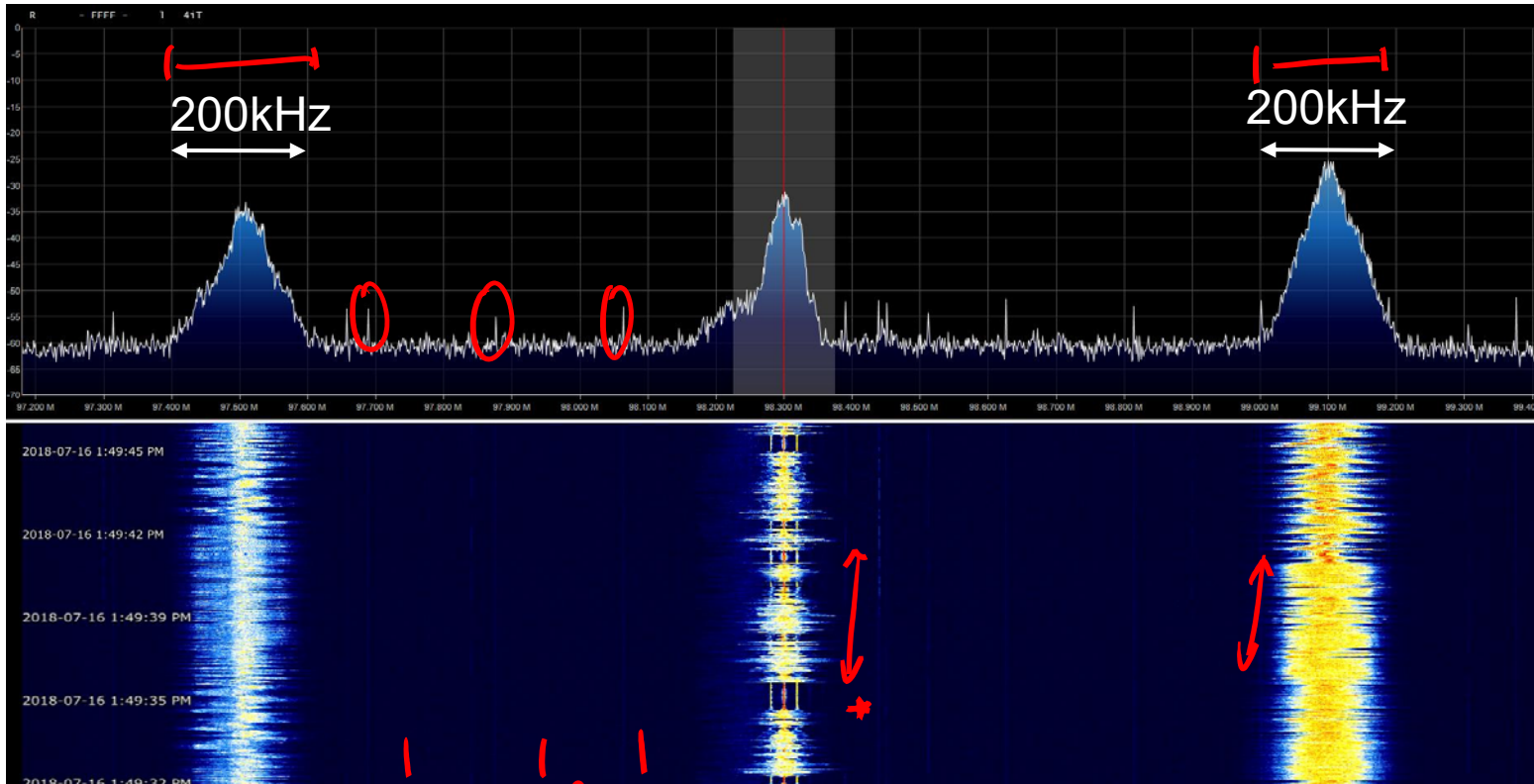
odd multiples of f_m

$J_n(x)$ is a Bessel Function of the first kind



Bandwidth defines channel spacing

Waterfall Chart



Example

KBCO

$$f_c = 97.3\text{MHz}$$

$$f_m = 20\text{ kHz}$$

$$f_{\Delta} = 200\text{ kHz}$$

97.1MHz

97.7

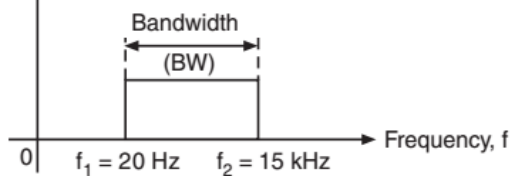
97.9

98.1

98.3MHz

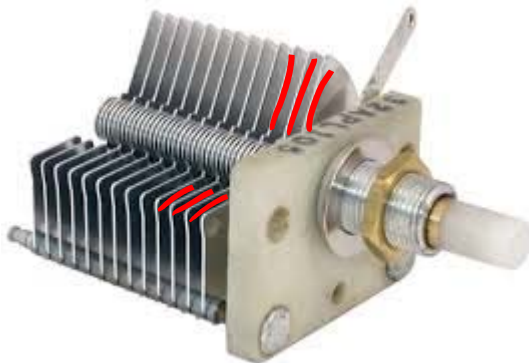
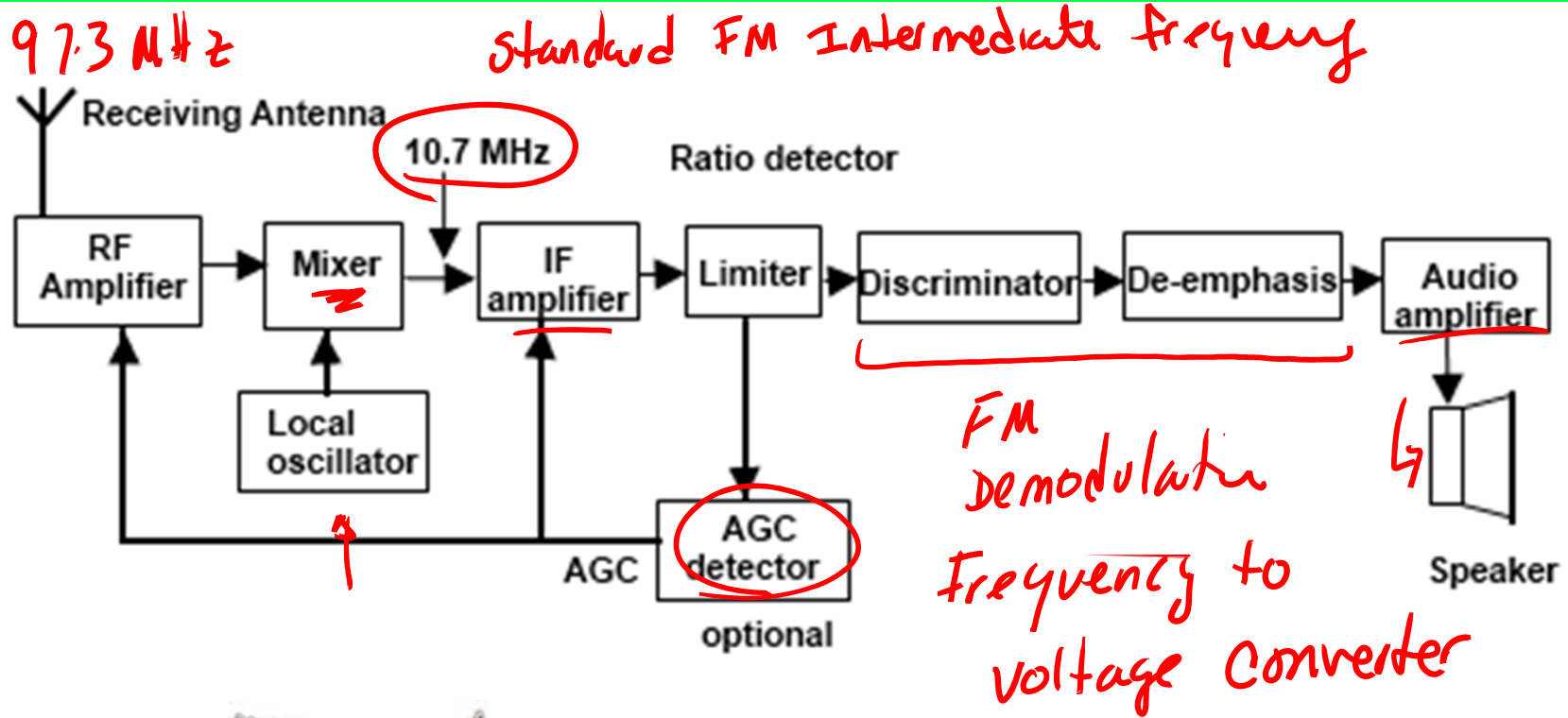
99.1MHz

Music Bandwidth



S.No.	Type of the signal	Range of frequency in Hz	Bandwidth in Hz
1.	Voice signal (speech) for telephony	300 – 3400	3,100
2.	Music signal	20 – 15000	14, 980
3.	TV signals (picture)	0 – 5 MHz	5 MHz
4.	Digital data	300 – 3400 (If it is using the telephone line for its transmission)	3,100

FM Receiver



Air variable capacitor used for tuning